Mean-Variance and Single-Index Model Portfolio Optimisation: Case in the Indonesian Stock Market

Adi Kurniawan Yusup*

ABSTRACT

Manuscript type: Research paper
Research aims: This study aims to compare the performance of mean-variance and single-index models in creating the optimal portfolio.
Design/Methodology/Approach: This study creates optimal portfolios using the mean-variance and single-index models with daily stock return data of 38 companies listed on the LQ45 index, IDX Composite index and Bank Indonesia's 7-Day (Reverse) Repo Rate from January 1, 2012 to December 31, 2019. The two models are compared using the Sharpe ratio.
Research findings: The result shows that the single-index model dominates the Indonesian Stock Exchange (IDX), more so than the mean-variance model. BBCA has the highest proportion for both mean-variance and single-index portfolios.
Theoretical contribution/Originality: This study compares two popular portfolio models in the Indonesian stock market.
Practitioner/Policy implication: This study helps investors to create optimal portfolios using a model that is more suited to the IDX.
Research limitation/Implication: This study creates the optimal portfolio without differentiating risk preferences (i.e., risk averse, risk moderate and risk taker). In addition, this research only uses daily return data and does not compare it with weekly and monthly data.

Keywords: Mean-variance Model, Optimal Portfolio, Single-index Model
JEL Classification: G11

* Corresponding authors. Adi Kurniawan Yusup is a doctoral degree student of Widya Mandala Catholic University, Surabaya, Indonesia, e-mail: adikurniawanyusup@hotmail.com

https://doi.org/10.22452/ajba.vol15no2.3

1. Introduction

The pandemic created significant problems in the business world. Most sectors worldwide, including in Indonesia, had to stop their operations temporarily to minimise the spread of Covid-19. The Central Bureau of Statistics of Indonesia reported that the country's GDP shrank by 5.32% in the second quarter of 2020, 3.49% in the third quarter and 2.19% in the last quarter (Bank Indonesia, 2020). The Indonesian government rolled out several policies to boost its GDP, including easy fiscal and monetary policies to increase consumption, investment, government spending, and export, as the four components of GDP (Aryusmar, 2020). As a result, many became interested in investing with the relaxation of investment policies. The Indonesia Stock Exchange (IDX) accounted for 3.88 million investors at the end of 2020 (Depository, 2020), and exceeded its annual target of 3 million last year.

Return is one of the factors that motivates investors as a reward for their courage in taking risks. Many investors invest in stocks to get a high return instantly (Hamka et al., 2020). They should use surplus funds after financing their primary need to invest because there is a risk of loss. Moreover, investing in stocks needs long periods to achieve high returns (Partono et al., 2017). However, Indonesia's Financial Service Authority (2019) showed that financial literacy in the country was at 38.03%, which is considered low. This is reflected in many using loans and primary-need money for investment. In addition, low financial literacy could increase the information asymmetry that increases herding behaviour (Din et al., 2021), that is, investing is the same stocks as others with the hope of high returns (Fransiska et al., 2018). Nevertheless, this behaviour causes investors to pick the wrong stocks for their portfolio, which leads to losses. Therefore, building an investment portfolio is a critical issue for investors.

Generally, investors are risk-averse (Reilly & Brown, 2003). They expect high returns with low risk, whereas investment theory states that there is a linear relationship between risk and return, i.e., high risk, high return (Rui et al., 2018). Hence, risk should be reduced to get the desired return with lower risk. A fundamental approach to minimising risk is diversification (Lee et al., 2020). Diversification is the allocation of a fixed portfolio across different types of securities and asset classes that limit exposure to any source of risk (Bodie et al., 2014). The optimal portfolio for the most returns with the lowest risk is located in the efficient frontier curve (Fama & French, 2004). However, creating an optimal portfolio is a challenge, especially for new investors. Investors need to work with a lot of information, e.g., historical price, trends and firm performance. They also need analytical tools to get the right composition of stocks in their portfolio.

One approach used to obtain an optimal portfolio is modern portfolio theory (MPT) or mean-variance analysis. MPT was established by Markowitz (1952), and attempts to maximise portfolio return for any given risk, or equivalently, minimise risk for any given amount of return. Both ways will produce the same result in creating an efficient portfolio. The basic concept behind MPT is that assets in a portfolio should not be viewed in isolation but should be evaluated by how it affects the portfolio's risk and return on the whole (Omisore et al., 2012). MPT seeks to reduce total variance as a component of risk of portfolio return by combining different assets with returns that are not positively correlated. This theory assumes that investors are rational and markets are efficient.

The Markowitz model provides several risk-return combinations which investors can choose from based on their preferences (Putra & Dana, 2020; Rigamonti, 2020). However, implementing the Markowitz model is very time-consuming, because it requires a lot of estimations to fill the covariance matrix. The model does not provide guidelines for forecasting the security risk premium, that is important for the construction of an efficient frontier of risky assets (Singh & Gautam, 2014). Sharpe (1963), in trying to simplify the Markowitz model, came up with a single-index model that reduces computational requirements and data. This model solves the portfolio optimisation problem by showing a linear relationship between risk and return—which is simpler to understand—and considers that the relationship between securities occurs only through their individual relationship with some indices of business activity. As a result, covariance data is reduced from $(n^2 - n)/2$ under the Markowitz model to only n measures of each security as it relates to the index (Chitnis, 2010). Nevertheless, single-index models may not be the most efficient with respect to volatility.

There are limited studies on comparing the mean-variance and single-index models in creating a portfolio. Furthermore, those studies show inconsistent results. Ozkan and Cakar (2020) show that the mean-variance model performs better in the markets of developed countries, while the single-index model performs better in developing countries. On the other hand, Chasanah et al. (2017), whose research sample is a developing country market, find that optimal portfolio formation with the mean-variance model is more dominant than the single-index model. Nyokangi (2016) shows that the choice between mean-variance and single-index models depends on the time span of study. On the other hand, Yuwono and Ramdhani (2017) find no significant return difference between the mean-variance model and single-index model.

Therefore, this paper aims to construct portfolios using the mean-variance and single-index models, and compare the performance of each in the IDX. The sample used in this research are stocks listed in the LQ45 index, the most liquid stocks in the IDX. This research is expected to give insight to investors—especially new investors—on constructing an optimal portfolio.

2. Literature Review

2.1 Portfolio Theory (Markowitz Theory)

Asset allocation for the sake of diversification in financial markets has become an important issue for investors. Investors expect asset allocation to give them the highest return with a particular risk, or the lowest with a certain expected return. Diversification is a risk reduction technique by spreading investment in different securities and asset classes (Bodie et al., 2014; Jayeola et al., 2017). In the traditional approach of diversification, the greater the number of securities invested in portfolio, the lower the risk (Lekovic, 2018). Based on the law of large numbers, this theory is supported by some its proponents, like Hicks (1935), Williams (1938), and Leavens (1945). Nevertheless, portfolio efficiency decreases as the number of portfolios increases because it results in excessive diversification. This can reduce portfolio risk, but ignores the correlation between returns on different assets (Francis & Kim, 2013).

Simple diversification was rejected by Markowitz (1952), who was awarded the Nobel Prize in 1990 for his essay "Portfolio Selection" and his book *Portfolio Selection: Efficient Diversification* (1959). He initiated the first quantitative theory of portfolio selection and management, currently known as MPT. This is a framework to create an investment portfolio based on the maximisation of expected returns and the simultaneous minimisation of investment risk (Fabozzi et al., 2002; Rodríguez et al., 2021). This theory uses return and risk in a coherent framework (Chen & Pan, 2013) and considers covariances among the stocks in the portfolio. This model is also called the mean-variance (MV) portfolio theory. Return is based on historical data while risk is obtained from standard deviation of returns. The Markowitz model provides a solution to overcome trade-off problems between risk and return in selecting a combination of assets in an optimal portfolio. The investor has a choice of either maximising return with any acceptable risk or to minimise risk with any acceptable level of return. Both options use the same mathematical process.

Efficient frontier refers to several alternative portfolios that give the highest return on certain risks or lowest risk on certain returns (Fabozzi et al., 2002; Letho et al., 2022). Nevertheless, not all portfolios on the efficient frontier are optimal. An optimal portfolio is located on the efficient frontier chosen by investors based on their individual preferences shown by utility. Theoretically, an optimal portfolio is defined as a tangency point between the efficient frontier and the investor's indifference curve, which presents the utility of the investor. A higher indifference curve has larger certainty equivalents and larger expected utility (N. Kumar, et al., 2014).

However, this theory has several limitations. Mean-variance portfolios find portfolio weight based on first moment and the parametric representation of second moment, assuming no predictable time variation. It assumes constant investment opportunities, and its static mean-variance solution is myopic and does not consider events beyond the present (Jones, 2017). MPT also assumes continuous pricing, a world in which markets are free, societies are stable, and investors are rational wealth maximisers (Curtis, 2004). In fact, this assumption is contradicted by observations of investors who get swept up in herd behaviour, whereby the investor tries to follow the decisions made by others and undervalues information available in the market (Bikhchandani & Sharma, 2000). They do not consider risk and return in choosing their portfolio. Moreover, these models need complex statistics-based mathematical modelling and formulas to support the concept and theoretical assumptions (Mangram, 2013). This is problematic when it is used to build portfolios from large stock groups, with many estimations needed for asset selection.

2.2 Single-index Model

The single-index model (SIM) concept was developed by Sharpe in 1963. This model simplifies Markowitz's mean-variance model (Mahmud, 2019) to overcome its complexity. This model is no longer based on inter-asset correlation, but instead on the assumption that co-movement between stock return is due to movement in market returns, or to be more specific, returns of a broad market index, R_M (Frankfurter et al., 1976; Hadiyoso et al., 2015). The single-index model specifies two sources of uncertainty for return: systematic and unique uncertainty. Systematic uncertainty, or beta (β), is uncertainty from the market or macroeconomics, while unique uncertainty, alpha (α), is uncertainty from the company or industry itself. Hence, a single-index model is written as follows:

$$R_i = a_i + b_i R_M + e$$

From the equation, the difference between expected return ($a_i + b_i R_M$) and actual return (R_i) is symbolised by e,, residual return. An important concept in the single-index model is β terminology. This refers to sensitivity towards market return and is usually predicted using historical data (B. R. Kumar & Fernandez, 2019). The higher the β, the more sensitive the stock is to market return. The basic assumption used in the single-index model is that stocks are correlated only if they have the same response toward market return. Therefore, the covariance between two stocks only can only be calculated based on the similarity of their response toward market returns.

The optimal portfolio of the single-index model is obtained by sorting the value of excess return to β (ERB) for each stock from the largest to the smallest (Elton et al., 1976). The negative ERB value or positive ERB that is caused by negative excess return and negative β has to be removed. Those ERB values are compared with the value of cut-off point of the shares (C^*). C^* is the largest value C_i of each individual stock. Stocks with larger ERB value compared to the C^* are included in the optimal portfolio and the weights are calculated, while others are taken out.

3. Methodology

3.1 Database

This research uses daily closing prices of stocks incorporated in the LQ45 index, IDX Composite index and Bank Indonesia's 7-Day (Reverse) Repo Rate (7-DRRR), from January 2012 to December 2019. LQ45 represents the 45 most liquid companies on the IDX based on the following criteria: highest capitalisation, huge transaction value and prospects for future growth (Malini, 2019). The annual closing price of stocks are gathered from Yahoo Finance, then matched with financial statement reports from IDX. The IHSG index is obtained from Yahoo Finance, and the risk-free rate date is obtained from Bank

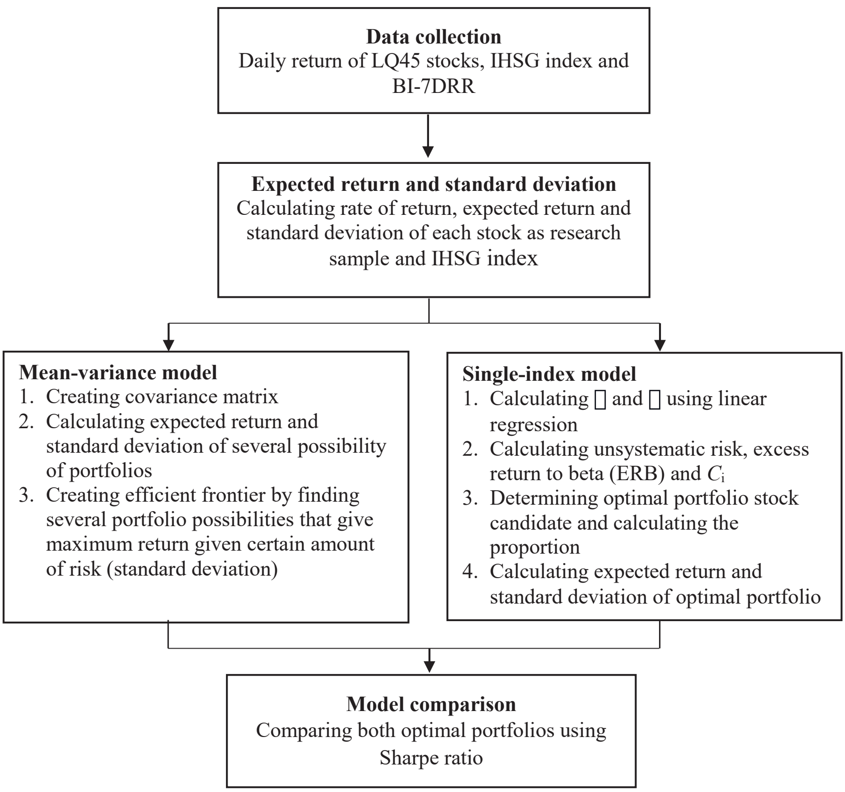
Indonesia's 7-DRRR report.

A total of 38 companies on the LQ45 index are used as the research sample. These are the only firms on the LQ45 index with enough information in the examined period. The other seven LQ45 companies do not have complete data—some issued stocks after 2012, and some had their stocks suspended, resulting in no trade for several days. When those stocks were traded again, the price could differ significantly from the last trading period, showing high volatility (Garcíaa et al., 2015). These 38 companies comprise various sectors, as listed in Appendix 1.

3.2 *Research Procedure*

Figure 1 shows the steps of constructing a portfolio using both the mean-variance and single-index models and comparing the results.

Figure 1: Research procedure



3.3 *Expected Return and Standard Deviation of Individual Stock*

The expected return of individual stock is the arithmetic mean of daily return during the research period (Benniga, 2006) as follows:

$$E(R_i) = \frac{\sum_{i=1}^{k} R_i}{k} \quad (1)$$

Where:

R_i	Rate of return of stock *i*
k	Number of rate of return data
$E(R_i)$	Expected return of stock *i*

Variances and standard deviation measure the risk of an individual stock. Variance measures the averaged squared difference between actual and average return, while standard deviation is the square root of variance (Bradford & Miller, 2009). Larger variances and standard deviation indicate greater volatility. Variance and standard deviation are calculated as follows:

$$\sigma_i^2 = \frac{\sum_{i=1}^{k} (R_i - E(R_i))}{k} \quad (2)$$

$$\sigma_i = \sqrt{\sigma_i^2} \quad (3)$$

Where:

R_i	Rate of return of stock *i*
k	Number of rate of return data
$E(R_i)$	Expected return of stock *i*
S_i^2	Variances of stock *i*
S_i	Standard deviation of stock *i*

3.4 *Mean-variance Model*

3.4.1 Creating Covariance Matrix

Covariance measures how returns on two risky assets move in tandem (Bodie et al., 2014). If the covariance is positive, two assets move together while they vary inversely if the covariance is negative. Covariance is calculated as follows:

$$\sigma_{ij} = \sum_{scenarios} \Pr(scenario)[R_i - E(R_i)][R_j - E(R_j)] \quad (4)$$

$$\text{Total covariance data} = \frac{n^2 - n}{2} \quad \sigma_i = \sqrt{\sigma_i^2} \quad (5)$$

Where:

R_i	Rate of return of stock *i*
$E(R_i)$	Expected return of stock *i*
R_j	Rate of return of stock *j*
S_{ij}	Covariance of stock *i* and *j*
n	Number of stocks as sample

The covariance matrix is a tool to calculate the standard deviation of a stock portfolio that is used to quantify risk. The format of the covariance matrix is as follows:

$$\text{Covariance matrix} = \begin{bmatrix} \mathrm{var}^2_{x_1} & \mathrm{cov}(x_1, x_2) & & \mathrm{cov}(x_1, x_n) \\ \mathrm{cov}(x_2, x_1) & \mathrm{var}^2_{x_2} & & \mathrm{cov}(x_2, x_n) \\ \cdots & & & \cdots \\ \mathrm{cov}(x_n, x_1) & \mathrm{cov}(x_n, x_2) & \ldots\ldots\ldots\ldots & \mathrm{var}^2_{x_n} \end{bmatrix} \quad (6)$$

3.4.2 Calculating Expected Return and Standard Deviation of Portfolio

The expected return of portfolio is the weighted average of individual stock return (Rachmat & Nugroho, 2013) and can be calculated using the following expression.

$$E(R_P) = \sum_{i=1}^{n} w_i E(R_i) \quad (7)$$

Where:

R_i	Rate of return of stock *i*
$E(R_p)$	Expected return on the portfolio
w_i	Weight of stock *i* in the portfolio
n	Number of stocks included in portfolio
$E(R_i)$	Expected return of individual stock *i*

Risk in portfolio management is measured through uncertainty of the returns. Correlation, coefficients, weights of each security and variance of its stocks are factors influencing portfolio risk (Aliu et al., 2017). Risk can be divided into two types: systematic risk (market risk or non-diversifiable risk) and unsystematic risk (diversifiable risk) (Wagdi & Tarek, 2019). Risk of the portfolio proxied by standard deviation can be calculated as follows:

$$\sigma^2_p = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{8}$$

$$\sigma_p = \sqrt{\sigma^2_p} \tag{9}$$

Where:

σ^2_p	Variance of the portfolio
w_i	Weight of stock *i* in the portfolio
w_j	Weight of stock *j* in the portfolio
n	Number of stocks included in portfolio
σ_{ij}	Covariance between stock *i* and stock *j*
σ_p	Standard deviation of portfolio

3.4.3 Creating Efficient Frontier Curve

An approach used in this research to find optimal portfolios located in efficient frontier curve is generating portfolio with maximum return for any given standard deviation.

$$\text{Max } E(R_P) = \sum_{i=1}^{n} w_i E(R_i) \tag{10}$$

Some restrictions used to find Max $E(R_p)$ are as follows:

Given standard deviation: $$\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}} \tag{11}$$

1. $$\sum_{i=1}^{n} w_i = 100\% \tag{12}$$
2. $$w_i \geq 0 \tag{13}$$

(no short position is available)

3.5 *Single-index Model*

3.5.1 Calculating α and β Using Linear Regression

To find α and β for each stock in single-index model, we regress the return of each stock on the market return. The regression equation is as follows:

$$R_i = a_i + b_i R_M + e \tag{14}$$

Where:

R_i	Return of a stock
R_M	Market return
α_i	Individual stock's alpha (intercept), part of return that is not affected by market
β_i	Individual stock's beta (slope), part of return that is affected by market
e	Residual

3.5.2 Calculating Unsystematic Risk, Excess Return to Beta (ERB) and Ci

Unsystematic risk is risk specific to individual company (firm-specific risk) (Masry & Menshawy, 2017). There are two methods to calculate unsystematic risk: direct and indirect. Using the direct method, unsystematic risk uses residuals of a factor model, such as CAPM and the Fama and French (1993) model. Using the indirect method, unsystematic risk can be calculated using the following formula:

Unsystematic risk = Total risk – systematic risk (15)

$$s^2(e_i) = s_i^2 - b_i^2 s_M^2 \tag{16}$$

Where:

$s^2(e_i)$	Unsystematic risk of stock *i*
s_i^2	Variances (total risk) of stock *i*
b_i^2	Beta square of stock *i*
s_M^2	Variances of market index

Excess return is the difference between stock return and the risk-free rate. In this case, the risk-free rate is 7-DRRR. Excess return to beta (ERB) measures the relative premium return of a stock toward unit risk that could not be diversified. ERB is calculated as follows:

$$ERB_i = \frac{E(R_i) - R_f}{\beta_i} \tag{17}$$

Where:

$E(R_i)$	Expected return of stock *i*
R_f	Risk-free rate
b_i	Beta of stock *i*
ERB_i	Excess return to beta of stock *i*

To determine which stocks are included in a portfolio, the cut-off point (C^*) has to be determined. The cut-off point value is obtained from the highest amount of C_i of each individual stock. C_i is calculated as follows:

$$C_i = \frac{\sigma_M^2 \sum_{i=1}^{n} \frac{[E(R_i) - R_f]\beta_i}{\sigma^2(e_i)}}{1 + \sigma_M^2 \sum_{i=1}^{n} \left(\frac{\beta_i^2}{\sigma^2(e_i)} \right)} \tag{18}$$

Where:

s_M^2	Variances of market
$E(R_i)$	Expected return of stock *i*
R_f	Risk-free rate
b_i	Beta of stock *i*
$S^2(e_i)$	Unsystematic risk of stock *i*

3.5.3 Determining Optimal Portfolio Stock Candidate and Calculating Proportion

The prerequisite of a stock included into an optimal portfolio is that its ERB value is higher than C* which can be written as follows:

$$ERB_i > C^* \tag{19}$$

Among all stocks that fulfil criteria (19), we have to calculate *z*-value to determine the stock weight in the optimal portfolio. The formula to calculate *z*-value with no short position available is as follows:

$$z_i = \frac{\beta_i}{\sigma^2(e_i)}(ERB_i - C^*) \tag{20}$$

Where:

Z_i	Z-value of stock *i*
β_i	Beta of stock *i*
$\sigma^2(e_i)$	Unsystematic risk of stock *i*
ERB_i	Excess return to beta of stock *i*
C^*	Cut-off point (maximum value of C_i)

Based on those *z*-values, the proportion of each stock in optimal portfolio can be calculated as:

$$w_i = \frac{z_i}{\sum_{i=1}^{n} z_i} \tag{21}$$

Where:

w_i	Weight of stock *i* in optimal portfolio
Z_i	S-value of stock *i*
n	Number of stocks that meet criteria (19)

3.5.4 Calculating Expected Return and Standard Deviation of Portfolio

The expected return of portfolio using the single-index model is calculated using the following equation:

$$E(R_p) = \alpha_p + \beta_p E(R_M) \tag{22}$$

Where:

$E(R_p)$	Expected return of portfolio
α_p	Portfolio alpha that is obtained from $\alpha_p = \sum_{i=1}^{n} w_i \alpha_i$
β_p	Portfolio beta that is obtained from $\beta_p = \sum_{i=1}^{n} w_i \beta_i$
$E(R_M)$	Expected market return

Portfolio risks consist of systematic risk and unsystematic risk. Therefore, total portfolio variance is the sum of systematic and unsystematic risk.

Portfolio risk = Systematic risk + Unsystematic risk

$$\sigma_P^2 = \beta_p^2 \sigma_M^2 + \sigma^2(e_p) \tag{23}$$

Where:

σ_P^2	Portfolio variance
β_p	Portfolio beta that is obtained from $\beta_p = \sum_{i=1}^{n} w_i \beta_i$
σ_M^2	Variances of market
$\sigma^2(e_p)$	Portfolio unsystematic risk that is obtained from $\sigma^2(e_p) = \sum_{i=1}^{n} w_i \sigma^2(e_i)$

3.6 Model Comparison

Two optimal portfolios, using the mean-variance and single-index models, with the same expected returns are compared using the Sharpe ratio. The Sharpe ratio is commonly used to measure portfolio performance (Zakamulin, 2008). This ratio is computed as the excess return relative to the risk-free rate divided by risk adjustment by using asset return volatility (Gatfaoui, 2009). The higher the Sharpe ratio is, the better performing the portfolio is.

$$\text{Sharpe ratio} = \text{Sharpe ratio} = \frac{E(R_p) - R_f}{\sigma_P} \quad (23)$$

Where:

$E(R_p)$	expected return of portfolio
R_F	risk-free rate
S_p	standard deviation of portfolio

Suppose the Sharpe ratio of mean-variance model is higher than single-index model. In that case, it is concluded that the mean-variance model dominates the Indonesian stock market more so than the single-index model. On the contrary, if the Sharpe ratio of the single-index model is higher than the mean-variance model, it dominates the IDX more than the mean-variance model.

4. Results

4.1 Expected Return and Standard Deviation

Table 1: Expected return, variances and standard deviation of individual stock and market

Securities	Expected return (%)	Variances (%)	Standard deviation (%)
ACES	0.098	0.064	25.343
ADRO	0.034	0.081	28.504
AKRA	0.041	0.053	23.123
ANTM	0.003	0.072	26.746
ASII	0.016	0.039	19.698
BBCA	0.085	0.022	14.739
BBNI	0.056	0.038	19.570
BBRI	0.080	0.039	19.665
BBTN	0.056	0.054	23.292
BMRI	0.062	0.038	19.426
BSDE	0.044	0.059	24.361
CPIN	0.101	0.084	29.049
CTRA	0.076	0.084	28.952
ERAA	0.089	0.117	34.250
EXCL	0.019	0.075	27.388
GGRM	0.013	0.043	20.687
ICBP	0.067	0.047	21.753
INCO	0.051	0.093	30.447
INDF	0.046	0.036	18.955
INKP	0.146	0.102	31.967
INTP	0.034	0.057	23.839
ITMG	-0.029	0.069	26.361
JPFA	0.081	0.088	29.617
JSMR	0.031	0.036	19.063
KLBF	0.064	0.039	19.739
MEDC	0.074	0.099	31.492
MNCN	0.055	0.087	29.452
PGAS	0.018	0.069	26.205
PTBA	0.022	0.073	27.011
PTPP	0.100	0.075	27.431
PWON	0.093	0.072	26.810
SMGR	0.031	0.052	22.907
SMRA	0.065	0.081	28.459
TBIG	0.075	0.052	22.701
TLKM	0.069	0.031	17.606
UNTR	0.019	0.057	23.817
UNVR	0.061	0.036	18.924
WIKA	1.023	18.480	429.887
Market Index	0.031	0.009	9.502

The first step to determine optimal portfolio using mean-variance and single models is calculating expected returns, variance and standard deviation of each stock. Table 1 presents daily expected return, standard deviation and variances of 38 LQ45 stocks and the IDX Composite.

Table 1 shows that 37 stocks have positive expected return, while one stock, ITMG, has negative expected return. The five shares with the highest expected return are WIKA with 1.02% returns, INKP with 0.15%, and CPIN, PTPP and ACES with 0.10. Meanwhile, stocks with higher risk, shown by the higher variances, are WIKA with 18.48% variance, ERAA with 0.12%, INKP and MEDC with 0.10%, and INCO, JPFA and MNCN with 0.09%.

Aside from that, the IDX Composite, as Indonesia's market index, shows a daily return 0.031%, a variance of 0.009% and standard deviation of 0.950%. The risk-free rate, calculated from the average of 7-DRRR during the research period, is 6.02% per year. Since this research uses daily return, the annual risk-free rate is divided by 241, the average number of days in a year for the research period. Hence, the risk-free rate used is 0.025%.

4.2 Mean-variance Model

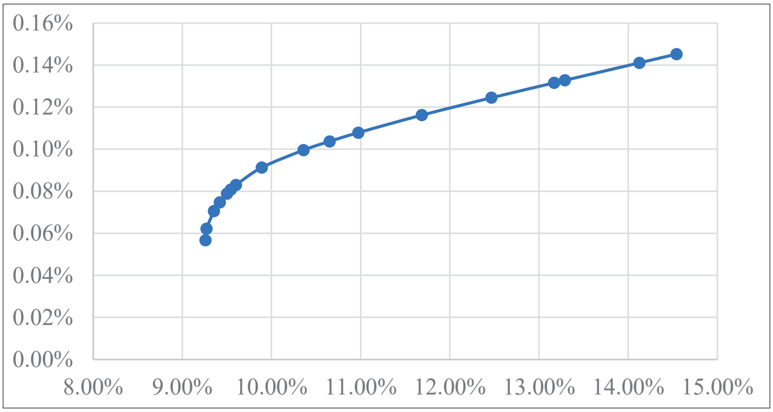
As the number of stocks analysed is 38 stocks, the number of covariance data that has to be calculated is $\frac{38^2 - 38}{2} = 703$ data. Based on these covariance data, the maximum return portfolio at any given desired risk level can be calculated. Table 2 shows 18 portfolio compositions along the efficient frontier line. 18 portfolios are generated using 38 available stocks and changing their weightings.

Portfolio 1 is the minimum-variance portfolio. This expected daily return of this portfolio is 0.056%, the variance is 0.856% and the standard deviation is 9.253%, with 26 stocks. BBCA has the highest proportion (18.99%) in this minimum-variance portfolio, as it is the least risky among the 38 stocks selected. The highest return per unit risk is shown by Portfolio 15. Its return to risk ratio is 0.99892, which means 0.99892% return obtained for every 1% risk borne.

As shown in Table 2, as the return is expected to increase, the number of stocks used to create portfolios with the lowest standard deviation decreases. To create portfolios 1 to 8, more than 20 stocks are selected. To create portfolios 9 to 13, between 10 and 20 stocks are selected, while for portfolio 14 to 18, less than 10 stocks are selected. This result has the same characteristics with a previous study conducted by Garcíaa et al. (2015). ACES, BBCA, INKP and WIKA are the four only stocks that are always selected in 18 efficient portfolios.

Investors can choose portfolios located in the efficient frontier because they are the best portfolios of all possible combinations (Halicki & Uphaus, 2014). Figure 2 shows the efficient frontier line using the 18 points in Table 2. Individual preferences and risk tolerance affect investment decisions (Lan et al., 2018). Those preferences are shown by the utility curve. The intersection between the utility curve and efficient frontier is the portfolio combination chosen by the investor.

Figure 2: Efficient frontier



Notes: Efficient portfolio = 18

Table 2: 18 efficient portfolio composition (%)

Portfolio	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
ACES	4.23	4.38	5.01	5.97	6.42	6.94	7.24	7.62	9.27	10.91	11.69	12.42	13.75	14.79	15.59	15.72	16.53	16.88
ADRO	1.77	1.88	1.76	1.76	1.67	1.56	1.42	1.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AKRA	4.30	4.41	3.92	3.43	3.10	2.71	2.41	1.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ANTM	2.98	3.13	2.32	1.48	0.92	0.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ASII	5.54	5.72	4.02	2.04	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BBCA	18.99	19.61	21.37	24.29	25.67	27.05	27.56	28.44	30.61	30.97	30.83	30.35	28.39	24.16	20.12	19.41	13.83	10.48
BBNI	2.99	3.21	3.10	3.15	3.00	2.80	2.65	2.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BBRI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BBTN	1.82	1.95	1.81	1.78	1.68	1.61	1.53	1.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BMRI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BSDE	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CPIN	0.00	0.00	0.00	0.31	0.80	1.34	1.62	1.98	3.53	4.93	5.58	6.33	8.08	10.17	11.92	12.22	14.23	15.23
CTRA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ERAA	0.00	0.00	0.06	0.09	0.71	0.96	1.10	1.28	1.94	2.20	2.29	2.38	2.46	2.26	2.03	1.99	1.63	1.40
EXCL	3.33	1.35	2.79	2.09	1.63	1.13	0.79	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GGRM	5.06	5.06	3.65	1.80	0.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ICBP	5.06	5.17	5.48	5.97	6.20	6.40	6.41	6.39	5.84	4.54	3.86	2.76	0.00	0.00	0.00	0.00	0.00	0.00
INCO	0.08	0.01	0.10	0.11	0.08	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
INDF	2.17	2.20	1.81	1.27	0.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
INKP	1.29	1.31	2.27	3.47	4.08	4.77	5.19	5.70	8.46	12.84	15.05	17.36	22.23	27.51	32.03	32.80	38.15	40.86
INTP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ITMG	2.91	2.94	1.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JPFA	1.41	1.40	2.01	2.69	2.89	3.10	3.17	3.22	3.23	3.02	2.92	2.71	1.97	0.47	0.00	0.00	0.00	0.00

Portfolio	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
JSMR	6.48	5.96	5.66	4.50	3.70	2.74	2.01	1.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KLBF	2.94	3.07	3.47	4.11	4.40	4.63	4.63	4.61	3.61	1.67	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MEDC	2.33	2.38	2.76	3.23	3.39	3.58	3.67	3.71	3.56	2.85	2.47	1.96	0.65	0.00	0.00	0.00	0.00	0.00
MNCN	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PGAS	1.05	1.14	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PTBA	0.78	0.82	0.91	1.01	0.69	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PTPP	0.00	0.00	0.00	0.42	0.75	1.10	1.28	1.55	2.67	4.28	5.09	5.84	7.02	7.49	7.63	7.63	7.49	7.27
PWON	0.00	0.00	0.00	0.00	0.28	0.67	0.87	1.12	2.28	3.78	4.53	5.21	6.11	6.13	5.94	5.89	5.29	4.77
SMGR	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SMRA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TBIG	7.88	7.92	8.35	8.90	9.14	9.38	9.49	9.58	9.68	9.50	9.39	9.11	7.88	5.10	2.44	1.97	0.00	0.00
TLKM	7.02	7.23	7.72	8.48	8.68	8.88	8.93	8.91	7.89	5.55	4.36	2.48	0.00	0.00	0.00	0.00	0.00	0.00
UNTR	0.96	1.06	0.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UNVR	6.57	6.65	6.99	7.48	7.67	7.82	7.76	7.61	5.79	2.18	0.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WIKA	0.05	0.05	0.10	0.17	0.20	0.24	0.26	0.30	0.49	0.78	0.93	1.10	1.47	1.91	2.30	2.37	2.85	3.10
Several calculations																		
$E(R_p)$	0.056	0.057	0.062	0.071	0.075	0.079	0.081	0.083	0.091	0.100	0.104	0.108	0.116	0.124	0.132	0.133	0.141	0.145
Variances (σ_p^2)	0.856	0.858	0.860	0.876	0.888	0.903	0.911	0.922	0.979	1.074	1.135	1.205	1.366	1.555	1.735	1.767	1.996	2.115
Standard deviation (σ_p)	9.253	9.262	9.273	9.357	9.421	9.502	9.545	9.603	9.893	10.362	10.654	10.975	11.686	12.469	13.171	13.293	14.127	14.545
Return/risk	0.602	0.612	0.671	0.754	0.793	0.830	0.846	0.864	0.923	0.961	0.974	0.983	0.994	0.998	0.999	0.999	0.999	0.999
Sharpe ratio	0.333	0.342	0.402	0.487	0.528	0.567	0.585	0.604	0.670	0.720	0.739	0.756	0.781	0.798	0.809	0.811	0.822	0.827
Number of stocks used	26	26	27	26	27	23	21	21	17	15	15	13	11	10	9	9	8	8

4.3 Single-index Model

The initial step to determine optimal portfolio using single-index model is calculating α and β using regression Equation 14. Using the result of α and β, unsystematic risk and excess return to beta (ERB) can be found by consecutively using Equations 16 and 17. Table 3 shows α, β, unsystematic risk and ERB of each 38 individual stocks.

Alpha shows unique stock return that is not affected by market return. The highest α is owned by WIKA, which is 0.01. β shows sensitivity of the stock toward market return. Positive β means an increase in market return will result in an increase in stock return. Meanwhile, a negative β means an increase in market return will result in a decrease in stock return. SMRA has the highest beta, 0.9131. This means when market return increases for 1%, SMRA's return will increase 0.9131%. Of 38 stocks analysed, only ASII has negative β during the research period. It means when market return increases, ASII return will decrease.

Unsystematic risk is risk that is related to particular stock or security. From Table 3, WIKA has the highest unsystematic risk of 0.1848. BBCA is stock that has the lowest unsystematic risk. This shows that most of its risk is affected by the market and it is good sign for the investor to invest in this company. ERB is the comparison between excess return and β. The highest ERB of these 38 stocks is CPIN, at 0.3819. Stocks that have negative or positive ERB due to negative excess return and negative beta have to be taken out from the list because they are ineligible for the establishment of an optimal portfolio. Of 38 stocks, eight stocks have to be taken out. The seven stocks with negative ERB are PTBA, UNTR, PGAS, GGRM, ITMG, ANTM and EXCL, while the one stock with positive ERB, negative β and negative excess return is ASII.

Table 3: Alpha, beta, unsystematic risk and ERB of 38 individual stocks

Stocks	Alpha (α)	Beta (β)	Unsystematic risk ($\sigma(e)$)	Excess return to beta (ERB)
ACES	0.0010	0.0050	0.0006	0.1466
ADRO	0.0003	0.0210	0.0008	0.0042
AKRA	0.0004	0.1146	0.0005	0.0014
ANTM	0.0000	0.0391	0.0007	-0.0055
ASII	0.0002	-0.0391	0.0004	0.0023
BBCA	0.0008	0.0252	0.0002	0.0238
BBNI	0.0005	0.0749	0.0004	0.0042
BBRI	0.0008	0.0949	0.0004	0.0058
BBTN	0.0005	0.0845	0.0005	0.0037
BMRI	0.0006	0.0570	0.0004	0.0065
BSDE	0.0004	0.0593	0.0006	0.0032
CPIN	0.0010	0.0020	0.0008	0.3819
CTRA	0.0007	0.0645	0.0008	0.0079
ERAA	0.0008	0.3275	0.0012	0.0019
EXCL	0.0002	0.0008	0.0008	-0.0768
GGRM	0.0000	0.5046	0.0004	-0.0002
ICBP	0.0005	0.4652	0.0005	0.0009
INCO	0.0003	0.5495	0.0009	0.0005
INDF	0.0003	0.5963	0.0003	0.0004
INKP	0.0013	0.5581	0.0010	0.0022
INTP	0.0001	0.8258	0.0005	0.0001
ITMG	-0.0004	0.3980	0.0007	-0.0014
JPFA	0.0006	0.6384	0.0008	0.0009
JSMR	0.0002	0.4752	0.0003	0.0001
KLBF	0.0005	0.5754	0.0004	0.0007
MEDC	0.0006	0.4025	0.0010	0.0012
MNCN	0.0003	0.8311	0.0008	0.0004
PGAS	0.0000	0.6132	0.0007	-0.0001
PTBA	0.0001	0.4816	0.0007	-0.0001
PTPP	0.0008	0.8098	0.0007	0.0009
PWON	0.0007	0.7888	0.0007	0.0009
SMGR	0.0001	0.7712	0.0005	0.0001
SMRA	0.0004	0.9131	0.0007	0.0004
TBIG	0.0007	0.3296	0.0005	0.0015
TLKM	0.0005	0.5168	0.0003	0.0009
UNTR	0.0000	0.5866	0.0005	-0.0001
UNVR	0.0004	0.6422	0.0003	0.0006
WIKA	0.0100	0.6309	0.1848	0.0158

The next step is calculating C_i of the remaining 30 stocks using Equation 18. Table 4 shows ERB, C_i and C^* for the remaining 30 stocks that fulfil the criteria. C^* is the highest value of individual C_i. From the table, the value of C^* is 0.00007, the value of C_i of PTPP, PWON and TLKM. A stock will be selected into optimal portfolio when its ERB value is higher than C^*. Based on its ERB value, all 30 stocks meet that requirement, so those 30 stocks will be included in optimal portfolio.

Table 4: ERB, C_i and ERB to C^* of 30 stocks

Stocks	ERB	C_i	ERB to C^*	Remarks
ACES	0.14662	0.00000	ERB > C^*	Portfolio candidate
ADRO	0.00423	0.00000	ERB > C^*	Portfolio candidate
AKRA	0.00136	0.00000	ERB > C^*	Portfolio candidate
BBCA	0.02383	0.00001	ERB > C^*	Portfolio candidate
BBNI	0.00419	0.00001	ERB > C^*	Portfolio candidate
BBRI	0.00582	0.00001	ERB > C^*	Portfolio candidate
BBTN	0.00368	0.00000	ERB > C^*	Portfolio candidate
BMRI	0.00646	0.00001	ERB > C^*	Portfolio candidate
BSDE	0.00320	0.00000	ERB > C^*	Portfolio candidate
CPIN	0.38185	0.00000	ERB > C^*	Portfolio candidate
CTRA	0.00787	0.00000	ERB > C^*	Portfolio candidate
ERAA	0.00194	0.00002	ERB > C^*	Portfolio candidate
ICBP	0.00091	0.00004	ERB > C^*	Portfolio candidate
INCO	0.00047	0.00001	ERB > C^*	Portfolio candidate
INDF	0.00035	0.00003	ERB > C^*	Portfolio candidate
INKP	0.00217	0.00006	ERB > C^*	Portfolio candidate
INTP	0.00011	0.00001	ERB > C^*	Portfolio candidate
JPFA	0.00088	0.00004	ERB > C^*	Portfolio candidate
JSMR	0.00013	0.00001	ERB > C^*	Portfolio candidate
KLBF	0.00068	0.00005	ERB > C^*	Portfolio candidate
MEDC	0.00122	0.00002	ERB > C^*	Portfolio candidate
MNCN	0.00036	0.00003	ERB > C^*	Portfolio candidate
PTPP	0.00093	0.00007	ERB > C^*	Portfolio candidate
PWON	0.00087	0.00007	ERB > C^*	Portfolio candidate
SMGR	0.00007	0.00001	ERB > C^*	Portfolio candidate
SMRA	0.00043	0.00004	ERB > C^*	Portfolio candidate
TBIG	0.00153	0.00003	ERB > C^*	Portfolio candidate
TLKM	0.00086	0.00007	ERB > C^*	Portfolio candidate
UNVR	0.00056	0.00006	ERB > C^*	Portfolio candidate
WIKA	0.01582	0.00000	ERB > C^*	Portfolio candidate
Maximum value (C^*)		0.00007		

Sharpe's single-index model not only identifies stocks that construct the portfolio, but also recommend the proportion of funds to be invested for each stock to reduce unsystematic risk and construct a highly diversified portfolio. Therefore, after finding stocks in an optimal portfolio, the weight of those stocks can be found by calculating the *z*-value by using Equation 20. The proportion of *z*-value of each individual stock compared to total *z*-value is the weight of the stock in the portfolio. Table 5 presents *z*-value and weight of each stock in the portfolio. Five stocks with the highest proportion in the single-index model portfolio are BBCA (12.860%), TLKM (6.641%), BBRI (6.590%), INKP (5.488%) and ACES (5.325%).

Table 5: Z-value and weight of 30 individual stocks in portfolio

Stocks	*z*-value	Weight (%)
ACES	1.142	5.325
ADRO	0.107	0.501
AKRA	0.276	1.287
BBCA	2.758	12.860
BBNI	0.807	3.763
BBRI	1.413	6.590
BBTN	0.563	2.624
BMRI	0.966	4.504
BSDE	0.313	1.458
CPIN	0.899	4.190
CTRA	0.600	2.797
ERAA	0.526	2.455
ICBP	0.856	3.993
INCO	0.244	1.138
INDF	0.505	2.357
INKP	1.177	5.488
INTP	0.066	0.308
JPFA	0.615	2.866
JSMR	0.073	0.342
KLBF	0.965	4.501
MEDC	0.472	2.200
MNCN	0.298	1.391
PTPP	1.000	4.662
PWON	0.943	4.399
SMGR	0.001	0.003
SMRA	0.448	2.091
TBIG	0.951	4.437
TLKM	1.424	6.641
UNVR	0.983	4.582
WIKA	0.054	0.251
Total	21.445	100.00

The last step is calculating expected return and standard deviation of portfolio that consists of 30 stocks whose weight has been determined. Before that, α, β and unsystematic risk for the portfolio have to be calculated. The α of the portfolio is the weighted average of individual stock α, and so too for β and unsystematic risk. Table 6 provides us the α, β and unsystematic risk of the portfolio.

Table 6: Alpha, beta and unsystematic risk of portfolio

α portfolio	β portfolio	σ (e) portfolio
0.000709	0.339544	0.001

The expected return of the portfolio is calculated using the single-index model regression equation.

$$E(R_p) = a_p + b_p E(R_M)$$
$$= 0.000709 + 0.3339544^{\prime}\ (0.031\%)$$
$$E(R_p) = 0.00081 = 0.081\%$$

The variance of the portfolio is calculated using Equation 23.

$$s_P^2 = b_p^2 s_M^2 + s^2(e_p)$$
$$= 0.339544^{2\,\prime}\ (0.009\%)^2 + 0.001$$
$$s_P^2 = 0.0010$$

The standard deviation is the square root of variance. Therefore, the standard deviation of the portfolio is:

$$\sqrt{0.0010} = 0.031623 = 3.162\%$$

4.4 Discussion

The mean-variance and single-index models result in portfolios that are optimal in their own way. The mean-variance model is more complicated than the single-index model. This section presents the performance comparison between the mean-variance and single-index model in the IDX. Both models are compared using the Sharpe ratio. The previous section shows that the expected return of portfolio using the single-index model is 0.081%, with a standard deviation of 3.162%. Referring to Table 2, mean-variance portfolio 7 has the same expected return of 0.081% with a standard deviation of 9.545%. In this case, with the same expected return, the standard deviation of the single-index model portfolio is lower than the mean-variance model portfolio. Table 7 presents a comparison of the five highest stocks, standard deviation and Sharpe ratio between the two models with the same expected return, 0.081%.

BBCA stock has the highest proportion in both models, with more than 20%. TLKM and ACES are in the top five stocks for both models. The Sharpe ratio calculated using Equation 24 with a 0.025% risk-free rate shows that the single-index model portfolio with a 1.77% ratio dominates the mean-variance model with 0.59%.

The study shows that the single-index model dominates the mean-variance model in creating the optimal portfolio. This finding is consistent with the findings of Ozkan and Cakar (2020), who state that the single-index model performs better in developing markets. This may happen because developing markets are more volatile than developed markets (Darby et al., 2019), and because the single-index model considers all aspects of the economy that avoid portfolio losses (Chanifah et al., 2020).

Table 7: Comparison between single-index model and mean-variance model

Component	Single-index Model	Mean-variance Model
Five highest stock composition	BBCA (12.860%)	BBCA (27.56%)
	TLKM (6.641%)	TBIG (9.49%)
	BBRI (6.590%)	TLKM (8.93%)
	INKP (5.488%)	UNVR (7.76%)
	ACES (5.325%)	ACES (7.24%)
Expected return	0.081%	0.081%
Standard deviation	3.162%	9.545%
Sharpe ratio	1.77%	0.59%

5. Conclusion

This research highlights the difficulties faced by new investors in creating optimal portfolios. Two well-known portfolio models that can be used are mean-variance and single-index. Both models offer investors the ability to create portfolios with maximum return on any desired level of risk, or minimum risk with any desired of level of return. However, the mean-variance model is more complex because it has a larger number of covariance data to process. In addition, this research shows that the single-index model dominates the Indonesian stock market compared to the mean-variance model.

With the same expected return, 0.081%, the single-index model provides a lower standard deviation of 3.162% compared to the mean-variance model, with 9.545%. This means the single-index model has a higher Sharpe ratio as a performance evaluation than the mean-variance model. The optimal portfolio using the single-index model consists of 30 stocks. Those stocks, sorted by proportion in portfolio, are BBCA, TLKM, BBRI, INKP, ACES, PTPP, UNVR, BMRI, KLBF, TBIG, PWON, CPIN, ICBP, BBNI, JPFA, CTRA, BBTN, ERAA, INDF, MEDC, SMRA, BSDE, MNCN, AKRA, INCO, ADRO, JSMR, INTP, WIKA and SMGR. BBCA is the stock with the highest proportion in the portfolio for both single-index and mean-variance models.

Based on our findings, several implications can be stated. The findings could help investors arrange optimal portfolios that benefit them. They can allocate their money to invest in several combination of stocks that maximise returns with a certain risk. The findings also expand previous portfolio literature with respect to the application of the single-index and mean-variance models in the Indonesian stock market. However, this study does not differ between risk preferences—i.e., risk averse, risk moderate and risk taker—and further research should be carried out to analyse optimal portfolios for each risk preference. Furthermore, further research can use other indices besides LQ45 and compare the results of daily, weekly or monthly closing data.

References

Aliu, F., Pavelkova, D., & Dehning, B. (2017). Portfolio risk-return analysis: The case of the automotive industry in the Czech Republic. *Journal of International Studies*, *10*(4), 72-83. https://doi.org/10.14254/2071-8330.2017/10-4/5

Aryusmar, D. (2020). The effect of the household consumption, investment, government expenditures and net exports on Indonesia's GDP in the Jokowi-JK era. *Journal of Critical Reviews*, *57*(8). https://doi.org/10.17762/pae.v57i8.1318

Bank Indonesia. (2020). Pertumbuhan ekonomi Indonesia triwulan IV 2020 melanjutkan perbaikan. https://www.bi.go.id/id/edukasi/Documents/Infografis-Pertumbuhan-Ekonomi-Indonesia-Triwulan-IV-2020.pdf

Benniga, S. (2006). *Principles of Finance with Excel*. Oxford: Oxford University Press.

Bikhchandani, S., & Sharma, S. (2000). Herd behavior in financial markets. *IMF Staff Papers*, *47*(3), 279-310. http://www.hozir.org/pars_docs/refs/252/251684/251684.pdf

Bodie, Z., Kane, A., & Marcus, A. J. (2014). *Investments* (*10th edition*). New York: McGraw Hill Education.

Bradford, J., & Miller, T., Jr. (2009). *A Brief History of Risk and Return. Fundamentals of Investments* (*5th edition*). New York: McGraw Hill.

Chanifah, S., Hamdani, & Gunawan, A. (2020). The comparison of applying single index model and capital asset pricing model by means achieving optimal portfolio. *Agregat: Jurnal Ekonomi dan Bisnis*, *4*(1). https://doi.org/10.22236/agregat_vol4/is1pp8-24

Chen, L., & Pan, H. (2013). A dynamic portfolio theory model based on minimum semi-absolute deviations criterion with an application in the Chinese stock markets. *China Finance Review International, 3*(3), 284-300. https://doi.org/10.1108/CFRI-05-2012-0052

Chitnis, A. (2010). Performance evaluation of two optimal portfolios by Sharpe's ratio. *Global Journal of Finance and Management*, *2*(1), 35-46.

Curtis, G. (2004). Modern portfolio theory and behavioral finance. *The Journal of Wealth Management*, *7*(2), 16-22. https://doi.org/10.3905/jwm.2004.434562

Darby, J., Zhang, H., & Zhang, J. (2019). Institutional trading in volatile markets: Evidence from Chinese stock markets. *Pacific-Basin Finance Journal*, *65*, 101484. https://doi.org/10.1016/j.pacfin.2020.101484

Din, S. M. U., Mehmood, S. K., Shahzad, A., Ahmad, I., Davidyants, A., & Abu-Rumman, A. (2021). The impact of behavioral biases on herding behavior of investors in Islamic financial products. *Frontiers in Psychology*, *11*. https://doi.org/10.3389/fpsyg.2020.600570

Elton, E. J., Gruber, M. J., & Padberg, M. W. (1976). Simple criteria for optimal portfolio selection. *Journal of Finance, 33*(1), 296-302.

Fabozzi, F., Gupta, F., & Markowitz, H. (2002). The legacy of modern portfolio theory. *Journal of Investing*, *11*(3), 7-22. https://doi.org/10.3905/joi.2002.319510

Fama, E. F., & French, K. R. (2004). The capital asset pricing model: Theory and evidence. *Journal of Economic Prespectives*, *18*, 25-46.

Francis, J. C., & Kim, D. (2013). *Modern Portfolio Theory: Foundations, Analysis and New Developments*. Hoboken: John Wiley & Sons.

Frankfurter, G. M., Phillips, H. E., & Seagle, J. P. (1976). Performance of the Sharpe portfolio selection model: A comparison. *Journal of Financial and Quantitative Analysis*, *11*(2), 195-204. https://doi.org/10.2307/2979049

Fransiska, M., Sumani, Willy, & Pangestu, S. (2018). Herding behavior in Indonesian investors. *International Research Journal of Business Studies*, *11*(2), 129-143. https://doi.org/10.21632/irjbs.11.2.129-143

Garcíaa, F., Bueno, J. A. G., & Oliver, J. (2015). Mean-variance investment strategy applied in emerging financial markets: Evidence from the Colombian stock market. *Intellectual Economics* 9, 22–29. http://doi.org/10.1016/j.intele.2015.09.003

Gatfaoui, H. (2009). Sharpe ratios and their fundamental components: An empirical study. SSRN. https://ssrn.com/abstract=1486073

Hadiyoso, A., Firdaus, M., & Sasongko, H. (2015). Building an optimal portfolio on Indonesia Sharia Stock Index (ISSI). *International Journal of Administrative Science and Organization*, *22*(2), 111-121. https://doi.org/10.20476/jbb.v22i2.5699

Halicki, M., & Uphaus, A. (2014). The efficient frontier and international portfolio diversification. *Zeszyty Naukowe Uniwersytetu Szczecińskiego nr 803, Finanse, Rynki Finansowe, Ubezpieczenia*" (66), 101-110.

Hamka, Halimah, H. S., Jupri, M., Budiono, R., & Tambi, A. M. (2020). The influence of financial literacy on interest in investing for the academic community of Akademi Keuangan & Bisnis Indonesia Internasional (AKBII), Bandung, Indonesia. *International Journal of Business, Economics and Social Development*, *1*(1), 1-12. https://doi.org/10.46336/ijbesd.v1i1.13

Indonesia Central Securities Depository (KSEI) (2020). KSEI awarded best central custodian and sub registry during its 24th anniversary. https://www.ksei.co.id/files/uploads/press_releases/press_file/en-us/158_ksei_awarded_as_best_custodian_in_southeast_asia_for_the_second_time_20190410114029.pdf

Jayeola, D., Ismail, Z., & Sufahani, S. (2017). Effects of diversification of assets in optimizing risk of portfolio. *Malaysian Journal of Fundamental and Applied Sciences*, *13*(4). https://doi.org/10.11113/mjfas.v0n0.567

Jones, C. K. (2017). Modern portfolio theory, digital portfolio theory and intertemporal portfolio choice. *American Journal of Industrial and Business Management*, *7*(7), 833-854. https://doi.org/10.4236/ajibm.2017.77059

Kumar, B. R., & Fernandez, M. (2019). Examination of index model and Prediction of beta: A case study examination in IT sector. *Accounting and Finance Research*, *8*, 226-231. https://doi.org/10.5430/afr.v8n2p226

Kumar, N., Singari, R. M., & Archana, S. (2014). Portfolio optimization: Indifference curve approach. *International Journal of Advance Research and Innovation*, *2*(1), 127-133.

Lan, Q., Xiong, Q., He, L., & Ma, C. (2018). Individual investment decision behaviors based on demographic characteristics: Case from China. *Plos One 13*(8): e0201916. https://doi.org/10.1371/journal.pone.0201916

Lee, Y., Kim, W. C., & Kim, J. H. (2020). Achieving portfolio diversification for individuals with low financial sustainability. *Sustainability 12*(17), 7073. https://doi.org/10.3390/su12177073

Lekovic, M. (2018). Investment diversification as a strategy for reducing investment risk. *Economic Horizons*, *20*(2), 169-184. https://doi.org/10.5937/ekonhor1802173L

Letho, L., Chelwa, G., & Alhassan, A. L. (2022). Cryptocurrencies and portfolio diversification in an emerging market. *China Finance Review International*, *12*(1), 20-50. https://doi.org/10.1108/CFRI-06-2021-0123

Mahmud, I. (2019). Optimal portfolio construction: Application of Sharpe's single-index model on Dhaka Stock Exchange. *Jurnal Ilmiah Bidang Akuntasi dan Manajemen*, *16*(1). http://doi.org/10.31106/jema.v16i1.1736

Malini, H. (2019). Efficient market hypothesis and market anomalies of LQ 45 Index in Indonesia Stock Exchange. *Sriwijaya International Journal and Dynamic Economics and Business*, *3*(2), 107-121. https://doi.org/10.29259/sijdeb.v3i2.107-121

Mangram, M. E. (2013). A simplified perspective of the Markowitz portfolio theory. *Global Journal of Business Research*, *7*, 59-70.

Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, *7*(1), 77-91.

Masry, M., & Menshawy, H. E. (2017). The impact of unsystematic risk on stock returns in an emerging capital market (ECM) country: An empirical study. *International Journal of Financial Research*, *9*(1), 189-202. https://doi.org/10.5430/ijfr.v9n1p189

Nyokangi, C. O. (2016). *Relative Performance of the Single Index Versus Mean Variance Optimization in Equity Portfolio Construction in Kenya* [Unpublished Master's dissertation]. Strathmore University, Nairobi, Kenya.

Omisore, I., Yusuf, M., & Christoper, N. (2012). The modern portfolio theory as an investment decision tool. *Journal of Accounting and Taxation*, *4*(2), March 2012, 19-28. https://doi.org/10.5897/JAT11.036

Otoritas Jasa Keuangan (2019). *Booklet Survei Nasional Literasi dan Inklusi Keuangan Tahun 2022*. https://www.ojk.go.id/id/berita-dan-kegiatan/info-terkini/Pages/Booklet-Survei-Nasional-Literasi-dan-Inklusi-Keuangan-Tahun-2022.aspx

Ozkan, O., & Cakar, R. (2020). Comparison of mean-variance and single index optimization methods in developed and developing markets. *Journal of the Faculty of Economics and Administrative Sciences*, *10*(1), 63-79. https://doi.org/10.18074/ckuiibfd.441098

Partono, T., Widiyanto, Yulianto, A., & Vidayanto, H. (2017). The analysis of optimal portfolio forming with single index model on Indonesian most trusted companies. *International Research Journal of Finance and Economics*, *163*. http://www.internationalresearchjournaloffinanceandeconomics.com/ISSUES/IRJFE_163_04.pdf

Putra, I. K. A. A. S., & Dana, I. M. (2020). Study of optimal portfolio performance comparison: Single index model and Markowitz model on LQ45 stocks in Indonesia stock exchange. *American Journal of Humanities and Social Sciences Research (AJHSSR)*, *4*(12), 237-244. https://www.ajhssr.com/wp-content/uploads/2020/12/ZE20412237244.pdf

Rachmat, D., & Nugroho, A. B. (2013). Portfolio determination and Markowitz efficient frontier in five Indonesian industrial sectors. *Journal of Business and Management*, *2*(1), 116-131. https://journal.sbm.itb.ac.id/index.php/jbm/article/viewFile/550/419

Reilly, F. K., & Brown, K. C. (2003). *Investment Analysis and Portofolio Management (7th edition)*. Hoboken: Wiley.

Rigamonti, A. (2020). Mean-variance optimization is a good choice, but for other reasons than you might think. *Risks*, *8*(1). https://doi.org/10.3390/risks8010029

Rodríguez, Y. E., Gómez, J. M., & Contreras, J. (2021). Diversified behavioral portfolio as an alternative to modern portfolio theory. *The North American Journal of Economics and Finance*, *58*, 101508. https://doi.org/10.1016/j.najef.2021.101508

Rui, K. X., Rasiah, D., Yen, Y. Y., Ramasamy, S., & Pillay, S. D. (2018). An analysis of the relationship between risk and expected return in Malaysia stock market: Test of the CAPM. *International Journal of Engineering & Technology*, *3*(21), 161-170. https://doi.org/10.14419/ijet.v7i3.21.17154

Sharpe, W. F. (1963). A simplified model for portfolio analysis. *Management Science*, *9*(2), 277-293.

Singh, S., & Gautam, J. (2014). The single index model and the construction of optimal portfolio: A case of banks listed on NSE India. *Risk Governance and Control: Financial Market and Institutions*, *4*(2), 110-115. https://doi.org/10.22495/rgcv4i2c1art3

Wagdi, O., & Tarek, Y. (2019). The impact of financial risk on systematic risks: International evidence. *Journal of Applied Finance & Banking*, *9*(6), 203-216.

Yuwono, T., & Ramdhani, D. (2017). Comparison analysis of portfolio using Markowitz model and single index model: Case in Jakarta Islamic Index. *Jounral of Multidisciplinary Academics*, *1*(1). https://www.kemalapublisher.com/index.php/JoMA/article/view/278/265

Zakamulin, V. (2008). Portfolio performance evaluation with generalized Sharpe ratios: Beyond the mean and variance. *Journal of Banking & Finance 33*(7), 1242-1254. https://doi.org/10.1016/j.jbankfin.2009.01.005