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The Role of Image Processing in Measuring Fractal Dimension

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ABSTRACT The characterization of irregular objects with fractal methods often leads to the estimation of the slope of a function plotted against a scale parameter. The slope is usually obtained with a linear regression with upper and lower boundaries that sets the acceptable range of fractal dimension value. This pattern is similar even when the variables are changed. In the present work, we developed a computer application to measure fractal dimension, with emphasis on medical images. The system has the ability to process the images involving the Red, Green, Blue (RGB) and the Hue, Saturation, (luminosity) Value (HSV) before any analysis of the image(s) is carried out. The results of the present work suggest a boundary (which determines how much image processing should be done in order to achieve relevant fractal dimension values) can be identified by systematically changing the parameter.

ABSTRAK Pencirian objek tidak sekata menggunakan kaedah fraktal akan menghasilkan anggaran kecerunan plot sesuatu fungsi melawan suatu parameter skala. Kecerunan graf diperolehi melalui kaedah regresi linear, di mana sempadan atas dan bawah akan menentukan julat nilai dimensi fraktal. Pola ini didapati konsisten untuk sebarang nilai pembolehubah. Kajian ini melibatkan pembangunan aplikasi komputer untuk menentukan dimensi fraktal dengan memberi penekanan kepada imej-imej perubatan. Sistem ini berupaya memproses imej-imej menggunakan julat 'Red, Green, Blue (RGB)' dan 'Hue, Saturation, Value (HSV)' sebelum analisis dilakukan ke atas imej-imej. Pemerhatian ke atas hasil dapatan mencadangkan wujudnya julat pemprosesan imej dan sempadannya boleh diperolehi melalui penukaran parameter secara sistematik.

(fractal, fractal dimension, image processing)

INTRODUCTION

The generalisation of dimensions to fractional values is not just a mathematical curiosity but has strong relevance to the real world [1], much like the concept of probability. The word 'fractal' is derived from the Latin root word, 'fragere' which means 'to break' [2, 3]. The basic building block of a fractal object is defined as an irregular form, which is an exact copy of the whole fractal object but on a different scale [4]. In other words, fractals are structures defined by irregularly shaped patterns that repeat themselves at different scales of space or in cycles over time within an object; or similar structures with a fraction of its original size. Throughout the years, the concept of fractals and fractal dimension has been used in various fields.

Unlike Euclidean shapes, fractals are often difficult to define precisely as the definition based solely on dimension is too narrow [5]. As it is, fractals are being analysed and studied based on the ability to see for one thing: a repeating of usually irregularly shaped pattern. Various studies incorporate fractal properties by relating the concept to different variables such as time and space [6].

In the present work, a computer application has been developed with the purpose of analysing medical images, via the calculation of fractal dimension values. Before the estimation (of fractal dimension value) can be done, a certain degree of tweaking and/or processing (of the images) is necessary for the purpose of achieving a kind of suitable 'environment' or characteristics in order to achieve relevant (fractal dimension) values [7-9]. The system has the capability to process the images based on the Hue-Saturation-Value (HSV) color definition and utilises the Box-Count Method (BCM) [10] to measure their fractal dimension values.

Using the Box-Count Method, a regular mesh grid is overlapped over a (fractal) structure and the amount of boxes containing the structure of an object being analysed is counted [10-12]. The value of the mesh size is denoted as s and the corresponding boxes which contain the structure is $N(s)$. The box size, s is decreased successively and the corresponding amount of boxes, $N(s)$ is counted. As the size of the box decreases, the boxes approximate the structure. That is,

$$N(\Delta s) = \text{constant} \times (1/\Delta s).$$

For complex structures,

$$N(\Delta s) = \text{constant} \times (1/\Delta s)^{-D},$$
$$\text{Log}\, N(\Delta s) = -D\, \text{Log}\, (1/\Delta s) + \text{c}.$$

Therefore the values of log $N(s)$ plotted versus log s shall produce a linear graph where the gradient of the line represents the fractal dimension value. The box counting method proposes a systematic measurement, which applies to any structures in a 2-dimensional plane and can readily be adapted for structures in a 3-dimensional plane. The method was chosen in this study based on its ease of use, the method is automatically computable, and applicable for patterns with or without self-similarity [7].

In the processing of the medical images, the system carried out the elimination of non-fractal and/or irrelevant objects in the images. Users shall do the image processing to eliminate 'distortions' in the images and to preserve the 'fractality' of the structures, so relevant information required to estimate the fractal dimension values of the structures can be retained as much as possible. This is done using the 'thresholding' method based on the HSV color definition [13]. The fractal dimension values of the (processed) medical images demonstrate the importance of the HSV property of the image processing capability of the system and how they influence (the estimation of) the fractal dimension value. It is also important to note that the changes were first recorded in the number of boxes calculated for each box sizes (before being translated into fractal dimension values), where finer points in an image can only be captured by smaller boxes whereas large, clear structures might render the smaller boxes irrelevant.

MATERIALS AND METHODS

In this work, a computer application has been developed on a modest system, as the transferability of the application is taken into consideration. All the tests to acquire data for this paper was carried out on the exact same system the application was developed on, to eliminate the probability of achieving inconsistent fractal dimension values [8]. The system consists of an Intel Pentium III processor (550 MHz), an Intel 82801AA main board, 256MB of SD RAM and on board 82810 System and Graphics Controller. Visual Basic 6.0 was utilised as the development software in the development of the application.

Fractal dimensional values for three linear fractals were estimated by the system in a reverse engineering method. This is done in order to enable a systematic comparison of the fractal dimension values with the predetermined values of the fractals. Linear fractal images are the outcome of absolute generating processes and the information relating to each step of the process can be calculated exactly [7]. That is, each step of the process follows a certain 'law', where the exact number of new structures which are identical with the original ones on an exact fraction of a scale are generated. The exact fractal dimension value (of the structure) can therefore be determined even after just two steps of the process [7].

The three structures discussed are the Sierpinski gasket, the Koch triadic curve and the Koch quadratic curve. An example of how the Sierpinski gasket is formed is shown in Figure 1. The fractal dimension, f_d for each images representing a particular iteration is calculated using the equation

$$f_d = \frac{\log(\text{no. of pieces})}{\log(\frac{1}{\text{scaling factor}})} = \frac{\log N(s)}{\log\left(\frac{1}{s}\right)}.$$

Similarly, the Koch triadic and quadratic curves started with a basic shape; the triangle and square, and then each lines are replaced with several smaller lines, each of which are a fraction of a scale of the original line. So that for the Koch triadic curve, each line of the basic triangle shown in Figure 2(a) shall be replaced with smaller lines, each of which is a fraction of a scale of the original line. Over several iterations, the basic triangle will finally form the Koch Triadic Curve, shown in Figure 2(b).

For the Koch quadratic curve, the basic square is rotated so that it follows a similar orientation to that of a diamond. From there, the square follows a set process of iteration in which each lines are replaced with 8 smaller lines, each of which is ¼ of the length of the original lines so that it finally turns into the pattern shown in Figure 3.

Although the comparison involving linear fractals will enable discussions using exact values, there is also a limitation to this method of comparison, in which the Hue and Saturation properties of the system will not be utilised. This is due to the fact that all the linear fractals are black and white in color, rendering the Hue and Saturation properties in the processing of their images irrelevant.

In order to fully utilize the image processing capabilities of the system, fractal dimension values of several medical images were estimated. As these images are not in black and white, the Hue and Saturation properties of the system were fully utilised. The images are pre-processed using an off the shelf image processing software; Adobe Photoshop 7.0, in order to achieve a 'controlled variable' environment:

(i) All images utilised have the same color orientation; where each image is within the purple-magenta range of colors.
(ii) The same image(s) in different color orientations; where two images are chosen and pre-processed to be in the purple-magenta, pink-blue and yellow-green range of colors.

RESULTS AND DISCUSSION

Image processing and fractal dimension estimation was done on the Koch Quadratic curve first, in order to see if any of the three HSV values should be given more priority than the others. Figure 4 illustrates the fractal dimension measurement of the Sierpinski Gasket using the system developed in the present work.

The results obtained confirmed that for black and white images, only the maximum (luminosity) 'Value' influenced the fractal dimension value. The following are the results of the fractal dimension estimation of the Koch Quadratic curve.

1. Fractal dimension value of 1.836 ± 0.306 was always obtained when the minimum Saturation value remained at 0.00 and maximum (luminosity) Value remained at 255, regardless of the change in the other values. This value (1.836 ± 0.306) corresponded with the condition of the processed image, which was observed to be a black patch, and the value obtained was the maximum fractal dimension value for this image.
2. Fractal dimension value of 0.000 ± 0.000 was always obtained when the minimum value of Saturation is increased (>0.00). Contrary to the condition of the processed image in (1), this corresponded with a totally blank screen. Put simply, the image became a white patch and the value obtained is actually the dimension value of blank space.
3. Fractal dimension value other than 1.836±0.306 or 0.000±0.000 was obtained only when the maximum (luminosity) Value of the image (of the Koch Quadratic Curve) is reduced (<255) and the minimum Saturation value remained

at 0.00. The fractal dimension value showed gradual decrement as the maximum (luminosity) value was decreased gradually. This goes to show that even for a black and white image, the processing of image within the HSV range (in this instance, only V is involved) influenced the fractal dimension value obtained.

For all the fractal dimension values in (3), the standard deviation is largest (between 11.8% to 26.5%) where a very small or very large reduction has been made to the image's maximum (luminosity) value. Since the change in fractal dimension value of black and white images was only recorded when the maximum (luminosity) value was being changed, this (very small or very large reduction to 'Value') was interpreted as too little or excessive image processing. The high standard deviation value suggested that too little or excessive image processing (in this instance the maximum 'Value') compromised the accuracy of the calculation of fractal dimension value.

For the three linear fractals evaluated, the nearest to the published fractal dimension value for the Koch Quadratic Curve was obtained when the maximum (luminosity) Value is between 135 and 150, while for the Koch Triadic Curve, maximum Value between 135 and 140 and for the Sierpinski Gasket, maximum Value between 135 and 145. Table 1 compares the published fractal dimension values of the three linear fractals and the values obtained in the present work.

Four medical images were taken into consideration in the process of determination of image processing boundaries (images acquired from www.gateway.ovid.com):

(1) Acid Fast Bacillus stain of a granuloma
(2) Acinic Carcinoma of Parotid Gland
(3) Acinic Carcinoma of Parotid Gland (enlarged)
(4) Acute Myelogenous Leukemia-bone marrow

These images were set to be within the same purple-magenta color orientation as a means to create a controlled environment. Table 2 presents the results for the images. For very high values of maximum HSV or very low values of minimum HSV (minimal image processing), the images become a black patch and achieved fractal dimension values of very near to two, whereas for very low values of maximum HSV or very high values of minimum HSV (excessive image processing), what remained are usually merely small dots with low fractal dimension values and/or with large deviation values.

Table 3 lists the fractal dimesions of 'Acinic Carcinoma of Parotid Gland' and 'enlarged Acinic Carcinoma of Parotid Gland' images that were set in different color orientation. The table shows that the HSV range is found to be dependent upon color orientation.

If the range is set properly, in every case it is possible to achieve very high fractal dimension values if there are only minimal image processing done, while very low fractal dimension values if the image processing is done excessively. This goes to show that there is a 'middle-range' where the HSV values should be set so that only a minimum amount of distortions of the image exist while retaining the maximum fractal features of the image. Users play a large part in tweaking the HSV range and determining whether or not the fractal features of an image is preserved. Figure 5 shows that for the same image, different fractal dimension values can be obtained by tweaking the HSV range.

CONCLUSION

Image processing (and the amount of which) is a crucial process before the estimation of a structure's fractal dimension can be done. Too little image processing will result in 'unfiltered', noisy image. In the context of medical images (and non-linear fractal structures in general), adequate image processing needs to be done as the 'patterns' in the images are more dispersed than that of linear fractals. Too much image processing, on the other hand, will result in a poor representation of the original image, where what remains are merely dots or a blank screen, making the fractal dimension estimation pointless.

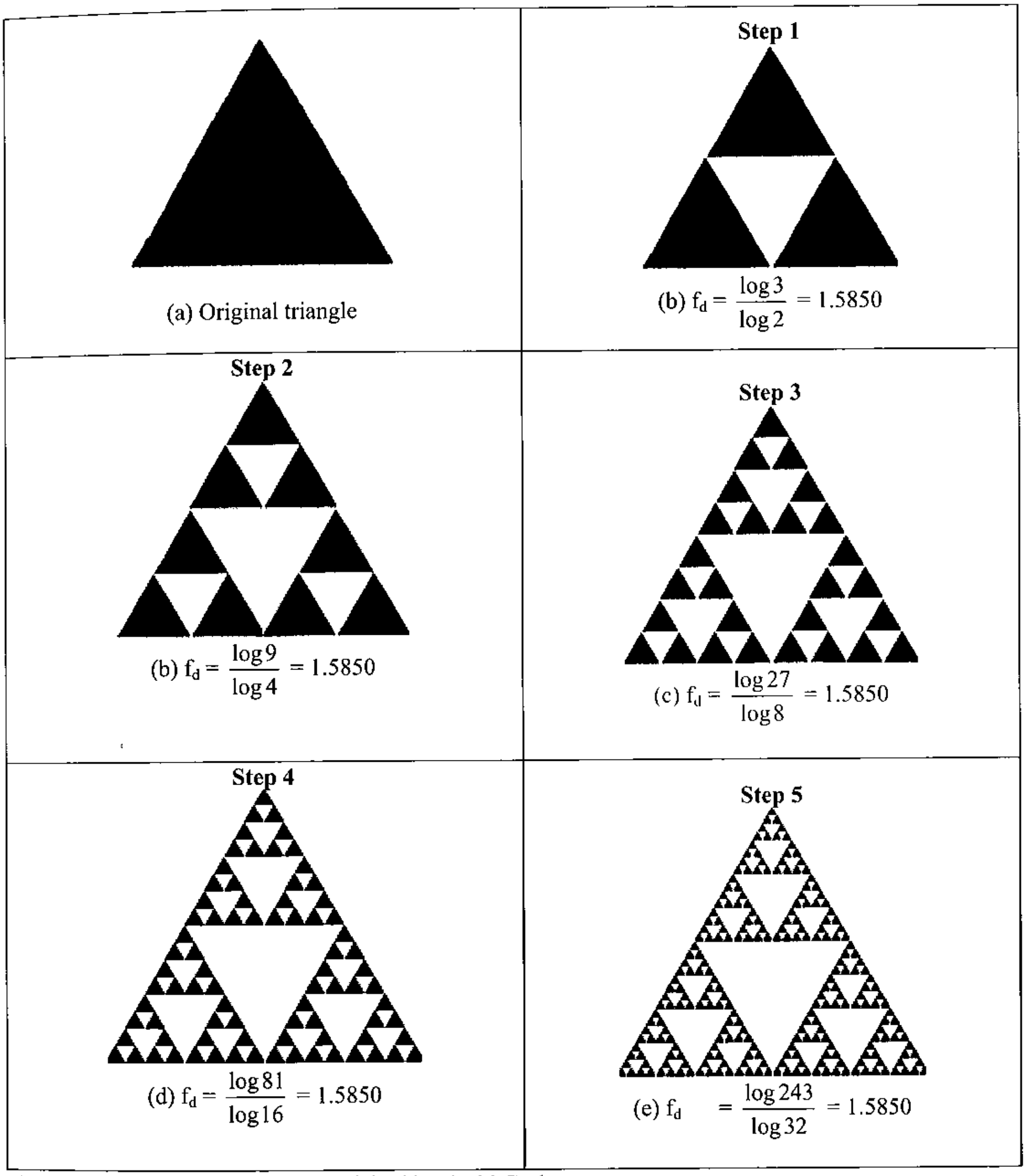

Figure 1. The Formation of the Sierpinski Gasket.

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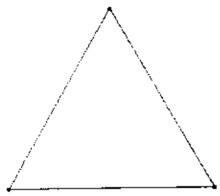
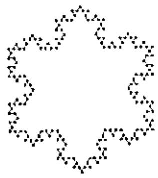


(a) Basic triangle



(b) The basic triangle, over several iterations, with $f_d = 1.2619$

Figure 2. The Koch Triadic Curve

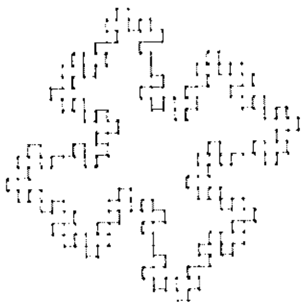


Figure 3. The Koch Quadratic Curve, $f_d = 1.5000$

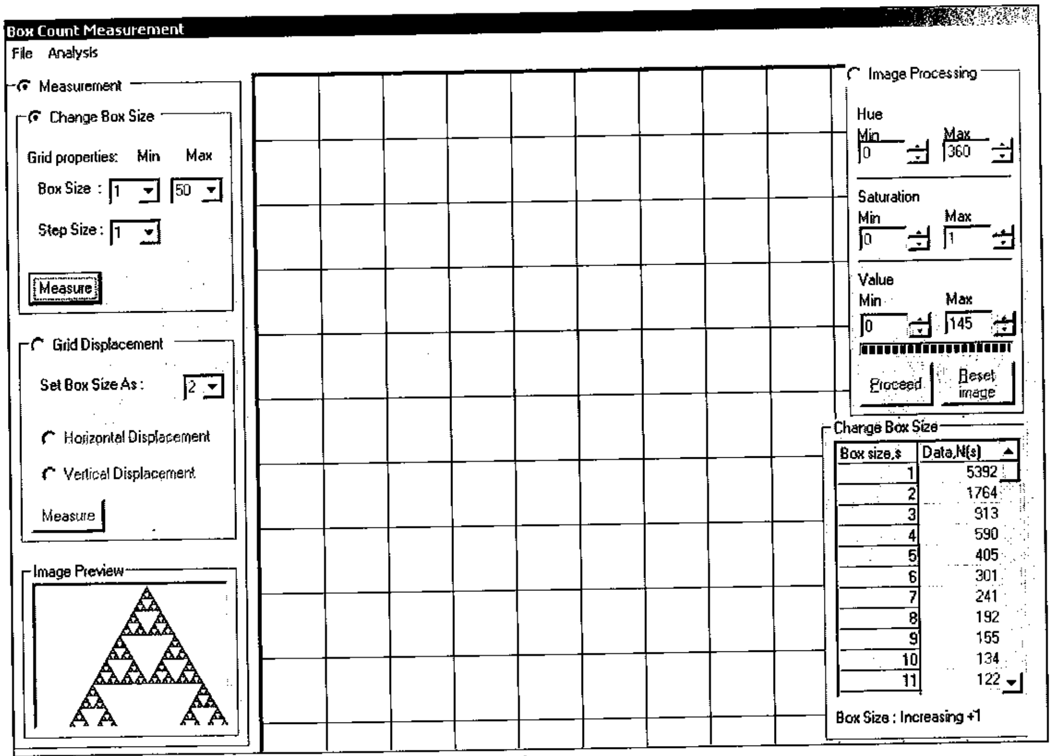


Figure 4. The Box-count Method commencing

Table 1. Comparison of the fractal dimensions between the published and calculated values

Structure	Published fractal dimension value [1]	Fractal dimension from the present work	% difference
Koch Quadratic Curve	1.5000	1.5008±0.118	0.05%
Koch Triadic Curve	1.2619	1.2665±0.059	0.36%
Sierpinski Gasket	1.5850	1.5853±0.079	0.02%

Table 2. The fractal dimensions of the medical images studied in this work.

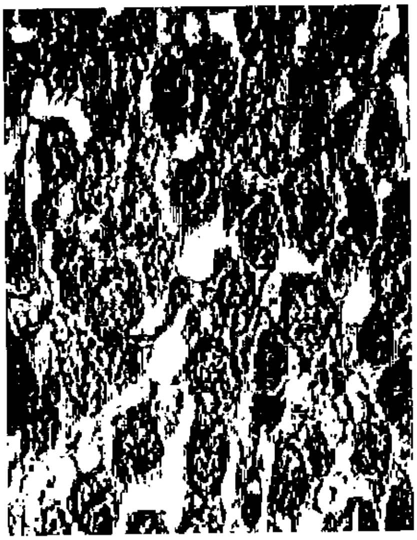
Image	Image processing		Fractal dimension value
Acid Fast Bacillus stain of a granuloma	Min Hue	305	1.439±0.142
		265	1.962±0.064
	Max Hue	280	1.337±0.169
		320	1.962±0.064
	Min Saturation	0.50	1.321±0.173
		0.30	1.962±0.064
	Max Saturation	0.35	1.439±0.151
		0.75	1.962±0.064
	Min Value	225	0.215±0.497
		185	1.962±0.064
	Max Value	185	0.000±0.000
		225	1.962±0.064
Acinic Carcinoma of Parotid Gland	Min Hue	305	1.304±0.356
		265	1.964±0.062
	Max Hue	280	1.657±0.289
		320	1.964±0.062
	Min Saturation	0.50	1.604±0.320
		0.30	1.964±0.062
	Max Saturation	0.25	1.533±0.180
		0.55	1.964±0.062
	Min Value	225	0.326±0.214
		185	1.964±0.062
	Max Value	185	0.000±0.000
		225	1.964±0.062

Acinic Carcinoma of Parotid Gland (enlarged)	Min Hue	305	1.493±0.162
		265	1.960±0.059
	Max Hue	280	1.036±0.147
		320	1.961±0.063
	Min Saturation	0.50	0.976±0.175
		0.30	1.961±0.063
	Max Saturation	0.25	1.150±0.189
		0.55	1.963±0.056
	Min Value	225	0.257±0.711
		185	1.961±0.063
	Max Value	185	0.000±0.000
		225	1.961±0.063
Acute Myelogenous Leukemia-bone marrow	Min Hue	305	1.266±0.123
		265	1.937±0.099
	Max Hue	280	1.664±0.129
		320	1.937±0.099
	Min Saturation	0.50	1.630±0.142
		0.30	1.937±0.099
	Max Saturation	0.35	1.523±0.179
		0.75	1.937±0.099
	Min Value	225	0.560±0.306
		185	1.937±0.099
	Max Value	185	0.000±0.000
		225	1.937±0.099

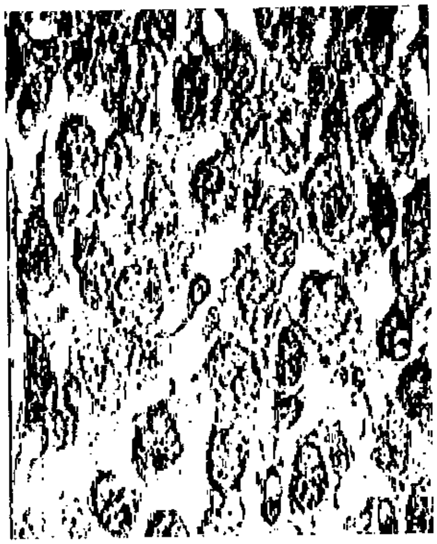
Table 3. Fractal dimesions for three different medical images set in different color orientation

Image	Image processing		Fractal dimension value
Acinic Carcinoma of Parotid Gland (pink-blue)	Min Hue	270	0.608±0.118
		230	1.964±0.062
	Max Hue	240	0.000±0.000
		280	1.964±0.062
	Min Saturation	0.60	1.849±0.165
		0.40	1.964±0.062
	Max Saturation	0.50	1.846±0.155
		0.70	1.964±0.062
	Min Value	250	0.833±0.405
		230	1.964±0.062
	Max Value	230	1.555±0.394
		250	1.964±0.062
Acinic Carcinoma of Parotid Gland	Min Hue	125	1.172±0.335
		85	1.964±0.062

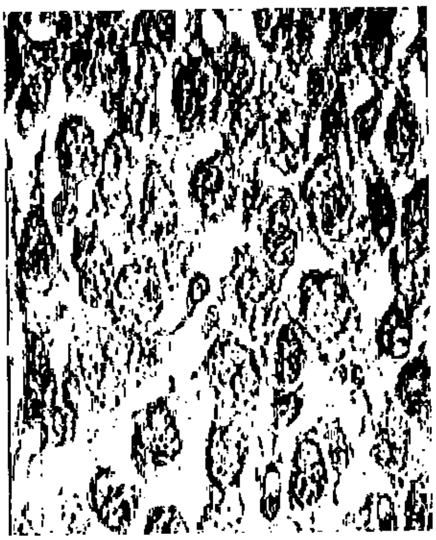
(yellow-green)	Max Hue	90	1.061±0.327
		130	1.964±0.062
	Min Saturation	0.50	1.518±0.348
		0.30	1.964±0.062
	Max Saturation	0.40	1.816±0.183
		0.60	1.964±0.062
	Min Value	200	1.521±0.534
		160	1.964±0.062
	Max Value	175	1.168±0.307
		215	1.964±0.062
Enlarged Acinic Carcinoma of Parotid Gland (pink-blue))	Min Hue	270	1.081±0.695
		230	1.961±0.063
	Max Hue	240	0.000±0.000
		280	1.962±0.056
	Min Saturation	0.60	1.861±0.084
		0.40	1.963±0.056
	Max Saturation	0.50	1.301±0.099
		0.70	1.961±0.063
	Min Value	250	0.248±0.199
		230	1.961±0.063
	Max Value	230	0.676±0.114
		250	1.961±0.063
Enlarged Acinic Carcinoma of Parotid Gland (yellow-green)	Min Hue	125	1.425±0.187
		85	1.963±0.056
	Max Hue	90	0.806±0.488
		130	1.961±0.063
	Min Saturation	0.50	1.680±0.143
		0.30	1.963±0.056
	Max Saturation	0.40	1.246±0.108
		0.60	1.961±0.063
	Min Value	200	1.115±0.115
		160	1.961±0.063
	Max Value	175	1.476±0.180
		215	1.963±0.056



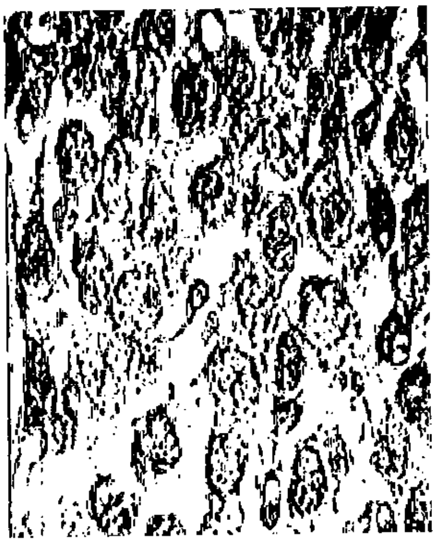
$54 < H < 308$
$0.09 < S < 0.79$
$5 < V < 177$
$f_d = 1.895 \pm 0.146$



$54 < H < 305$
$0.12 < S < 0.70$
$10 < V < 170$
$f_d = 1.805 \pm 0.081$



$54 < H < 305$
$0.12 < S < 0.69$
$10 < V < 170$
$f_d = 1.804 \pm 0.082$



$54 < H < 304$
$0.12 < S < 0.69$
$10 < V < 170$
$f_d = 1.789 \pm 0.103$

Figure 5. Images of Acute Myelogenous Leukemia-bone marrow

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