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EFFECTIVENESS OF MAYER'S PROBLEM SOLVING MODEL WITH VISUAL REPRESENTATION TEACHING STRATEGY IN ENHANCING YEAR FOUR PUPILS' MATHEMATICAL PROBLEM SOLVING ABILITY

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ABSTRACT

Ability of solving mathematical non-routine problems among Malaysian students remains at a low level, therefore there is a need to introduce a new and effective teaching strategy for the teaching and learning of mathematical problem solving. This study aims to determine the effectiveness of Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy in enhancing Year 4 students' mathematical problem solving ability. A sample of 175 Year 4 students were drawn using a convenient sampling technique with two classes assigned as experimental groups namely Mayer's problem solving Model (MM) group (n = 57) and MMVR group (n = 58), and the other one as the control group (n = 60). The MM group was given Mayer's problem solving Model (MM) teaching strategy treatment, while the MMVR group was given Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy treatment, respectively. The control group on the other hand did not undergo any additional intervention from researcher. The results of one-way ANCOVA analysis reveals that there is a statistically significant difference in the mean of the posttest score in MPSAT between the three groups [$F(2, 171) = 291.44$, $p < .0005$], with the adjusted means of posttest scores of MPSAT for MM, MMVR and Control groups were 105.84 (SE = .94), 116.14 (SE = .93), and 85.94 (SE = .89) respectively. The results suggest that mathematics teachers can modify their current pedagogy following Mayer's problem solving Model with visual representation teaching strategy to increase students' performance in problem solving.

Keywords: *Mathematical Problem Solving Ability, Mayer's Problem Solving Model, Visual Representation*

INTRODUCTION

Problem solving plays a prominent role in contemporary mathematics education. The need for learners to become successful problem solvers has become a dominant theme in many national standards (NCSS, 1997; NCTE, 1996; NCTM, 1989, 1991, 2000, & 2003). Aydogdu and Ayaz (2015) states that problem solving contributes to mathematics itself and it is the centre of the mathematics curriculum. Solving a problem requires students to think critically when deciding and developing their own strategy based on what they learnt and developed in previous problems, where algorithm cannot be directly applied. National Education Association (NEA, 2002) mentioned that today's citizens must be active critical thinkers if they are to compare evidence, evaluate competing claims, and make sensible decisions. However, from the results of PISA and TIMSS, it has been noted that the ability of solving mathematical problems among Malaysian students remains at a low level (OECD, 2012; Stephen et al., 2016).

Teachers often incorporated problem solving in the mathematics classroom by using different approaches (Van de Walle, Karp, & Bay-Williams, 2010). Siswono (2008) reported that mathematics learning process is still going on conventionally/ traditionally and tends to be mechanistic. It means that students listen, imitate or copy exactly the same way what the teacher gives without initiative. On the other hand, findings from the Teacher Education and Development Study in Mathematics (TEDS-M-M) states that pre-service teachers have difficulty solving abstract problems and problems requiring multiple steps (Tatto et al., 2011). Unal (2017) felt that there should be new methods of teaching and learning. Teaching problem solving is not only about providing a model and real problems to students, but also about the guidance of the teacher (Jose, 2017).

Successes in problem solving and achievement measures are influenced by the degree to which students are supported to gain facility with representations as problem representation is crucial to effective problem solving (Krawec, 2014). The literature about teachers' understanding and competence with visual representations is relatively found to be little. Beginning elementary school teachers are frequently have difficulty in selecting and employing visual representations such as number lines and hundred squares, and that their selections are based on superficial attractiveness rather than the effectiveness of the representations for the mathematics they want children to learn (Turner, 2008). Dreher and Kuntze (2015) states that even secondary school mathematics teachers do not fully understand the role and use of different forms of visual representations for learning about and teaching fractions. Also, Yew, Norul, and Syed (2016) mentioned that pre-service teachers who were unable to solve the problem correctly seemed to use limited and incorrect mathematical terminology, lack understanding of the problem, and were unable to make representations of the word problems.

Alternatively, Mayer's problem solving model is believed to enhance problem solving ability among students (Mayer, 1985). Mayer's problem solving model has visual representation component which past studies proved that it is an effective strategy to improve students' problem solving ability. However, this model not fully employing visual representation strategy throughout the model as only Mayer's first stage of problem representation converts a problem from words into an internal representation to an external representation (Mayer, 1985). This is insufficient to solve a problem accurately because, the more the visual representations include appropriate relational and numerical components, the closer they would fall on the accurate solution of the problem (Krawec, 2014). Therefore, this study employs visual representation into every stage of Mayer's problem solving model in order to enhance students' problem solving ability. The main objective of this study is to investigate the effectiveness of Mayer's problem solving Model with visual representation (MMVR) teaching strategy in enhancing Year Four students' mathematical problem solving ability. This study aimed at addressing the following research question:

1. Is there any significant difference in the mathematical problem solving ability of Year 4 students in MM group, MMVR group and control group after the treatments, after controlling the pretest score?

This research has some limitations. The generalizability of the results to a larger population might be limited due to the lack of random assignment into test groups which leads to non-equivalent test groups.

Mathematical Problem Solving Ability

Developing problem solving ability among school children has long been a persistent goal of mathematics education community for over a century. The mathematics education has changed fundamentally from an emphasis on knowledge and procedural skills to a focus on the active process of extending and applying known concepts in new contexts and problem solving (Schoenfeld, 2008). A growing interest in teaching mathematics in outdoor setting is a recent trend in Swedish elementary schools (Milrad, 2010). Teachers believe that this particular approach motivates the children more than solving problems in textbooks, thereby providing new ways to introduce and work with mathematical problem solving.

The National Council of Teachers of Mathematics suggested that mathematics teaching is based on problem solving because, they say, it incorporates skills and functions which are an important part of daily life (NCTM, 1980). Furthermore, it can help people to adapt to changes and unexpected problems in their careers and other aspects of their lives. More recently the Council endorsed this suggestion with the statement that problem solving should underlie all aspects of mathematics teaching in order to give students experience of the power of mathematics in the world around them. They see problem solving as a tool for students to develop, evaluate and refine their own theories about mathematics and the theories of others.

There are many interpretations about problem solving ability in mathematics. Among these, Polya's opinion is the most referred by many maths observers (Apulina & Surya, 2017). Polya used to be quick to point out that students need help to develop problem solving ability (understand the problem, make a plan, carry out the plan, and look backwards) and it needs to be taught correctly by teachers. Therefore, the problem solving ability measured in this study is based on Polya's problem solving process namely understand the problem, make a plan, carry out the plan, and look backwards.

Polya's Problem Solving Model

George Polya, the founder of modern-day theory in mathematical problem solving, developed an exact treatise on familiar heuristics for solving mathematical problems in his 1945 book titled *How To Solve It* (Polya, 1945).

Polya describes there are four steps in solving the problem, namely: (1) understand the problem: in this activity is to formulate: what is known, what is asked whether the information sufficient, condition (condition of) what should meet, restate the original problem in a more operational (solvable) way. (2) planning the solution: the activities carried out in this step is trying to find or recall issues you have solved that has similarities with the properties that will be solved, look for patterns or rules, draw up resolution procedures. (3) carry out the plan: the activities in this step are performed the procedures that have been created in the previous step to the settlement. (4) to look back the procedures and results of the settlement: activities in this step is analysing and evaluating whether the procedures applied, and the results obtained are correct, whether there are other procedures that are more effective, whether procedures have created can be used to solve similar problems, or whether the procedures generalizations can be made.

In distinction to conventional mathematics classroom environments, Polya's problem solving process offers students with opportunities to enhance their capabilities to adapt and change methods to fit new situations. Furthermore, students taking part in learning mathematical procedures related with communication, representation, modelling and reasoning.

Mayer's Problem Solving Model

Richard Mayer made significant contributions to problem solving beginning in the 1980s. Mayer (1983) viewed problem solving as a complex, multi-step cognitive system which requires one to associate preceding experiences to the problem at hand and further act upon the solution. He argued that a problem had to be paraphrased, comprehensively understood, and then visually integrated into a theoretically correct and complete schematic structure in order to reach the solution.

Mayer (1985) proposed a problem solving model that explains the problem-solving process which occurs in four stages specifically problem translation and problem integration (student' representation of the problem); and solution planning, and solution execution (specific strategies used in the problem). The indispensable problem-solving process requires students to first acquire the meaning of the problem and implications of the text. Next, the student develops an appropriate representation of the problem. Finally, the student links this representation to the best strategy for solving the problem (Mayer, 1985).

Richard Mayer has made enormous contributions to word problem solving using representation. Further expanding theory on a schema, Mayer confirmed that students do compare problems at hand to the schema for previously solved problems (Mayer, 1985). Furthermore, when students lack a schema for a problem they are facing, the students' representation of the problem is far more likely to be incorrect (Mayer, 1983). Incorrect representation of a problem is likely to produce an incorrect solution. In contrast, Mayer points out the fact that typical problem-solving instruction tends to focus on facts and algorithms rather than on correct representation (Mayer, 1989).

Visual Representation and Mathematical Problem Solving

Representation is one of the process standard which should enable students to know and do from kindergarten to K-12 (Istadi, Kusmayadi, & Sujadi, 2017). Representations can be expressed in the form of visual, verbal, and symbolic. Some benefits of representations could motivate students' mathematical ideas, especially in problem solving ability (NCTM, 2000; Sajadi, Parvaneh, & Rostamy-Malkhalifeh, 2013; Yee & Bostic, 2014). Besides in solving the problem, representations are useful in understanding abstract mathematical concepts. For instance, in the transition between arithmetic and algebra by geometric representations as well as second-degree polynomials in teaching factoring (Cabahug, 2012; Panasuk & Beyranevand, 2011).

Representing information visually is seen an efficient representation process in mathematics education, especially in problem solving (Guler & Ciltas, 2011). The importance of using visual representations in mathematics education can be seen by the contribution it makes to the development of understanding and intuitional perspectives. Using visual representations in the problem solving process may not always be effective and in some situations it may even lead to incorrect solutions (Guler & Ciltas, 2011), however creating visual representations that emphasize spatial relationships in the process of solving mathematical problems can contribute to problem solving success. Kilis, Uzun, and Technology (2018) mentioned that being able to visualize the problem in one's mind is a reason behind correctly solving the problem.

Solving word problem seems to be very difficult if the students are unable to make a relation between the known and unknown, particularly when the student faced difficulties in understanding the problem text (Boonen, van der Schoot, van Wesel, de Vries, & Jolles, 2013). The comprehension of the student can also refine by use of visualization to simulate the student thinking varies rather than focusing on symbolism and formalism approach (Lavy, 2007). The effective tools in learning mathematics are through visuals, which provide an alternative mass resource almost throughout the media as the representation of the simplified version of mathematical language, particularly in delivering the process of solving problem (Lavy, 2007).

METHODOLOGY

Research Design and Participants

A quasi-experimental non-equivalent pre-test and post-test design was used in this study. This design consists of three groups of respondents namely, experimental group 1, experimental group 2, and control group. The experimental group 1 namely MM group, learned mathematical problem solving using Mayer's Problem Solving Model (MM) only, the experimental group 2 namely MMVR group, learned mathematical problem solving using Mayer's Problem Solving Model with Visual Representation (MMVR) teaching strategy, and the control group learned mathematical problem solving without using Mayer's Problem Solving Model (MM) and Mayer's Problem Solving Model with Visual Representation (MMVR) teaching strategy. Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy, required students to use both MM and Visual Representation (VR) at the same time in order to examine whether the use of MM and VR can enhance students' problem solving ability or not.

The sample of this study was selected through convenience sampling where participants were Year four students selected from private school in Klang Valley district. 57 (32.6%) students were in MM group, 58 (33.1%) students were in MMVR group, and 60 (34.3%) students were in control group. Since all the sample is mix abilities students therefore researcher randomly assigned samples into three groups. Table 1 below shows the distribution of samples according to groups.

Table 1
Sample Distribution

Group	Number of students
MM	57 (32.6%)
MMVR	58 (33.1%)
Control	60 (34.3%)
Total	175

Treatments

Two treatments were involved in order to answer the research question for this study. The experimental 1 group students (n = 57) and experimental 2 group students (n = 58) underwent mathematics lesson during school hours as normal, together with extra instructions which were Mayer's problem solving Model (MM) and Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy respectively, after the school hours. Whereas, the control group students (N = 60), underwent mathematics lesson during school hours as normal, without any additional instruction after school hours.

The first treatment, which is called as Mayer's problem solving Model (MM) teaching strategy, was intended to provide the instructional assistance based on Mayer's (1985) four step problem solving model. The MM teaching strategy can help students decide and what to do when solving mathematical problems. Students learn how to translate the mathematical problems, integrated the information presented, developed logical plans to solve problems, and carried out the plans in an appropriate manner. The second treatment, Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy, required students to use both MM and Visual Representation (VR) at the same time in order to examine whether the use of MM and VR can enhance students' problem solving ability or not.

The sub-dimension problem solving abilities derived from Polya's problem solving model, namely: understand the problem ability, devise a plan ability, and carry out the plan ability was examined in every ten consecutive sessions during the treatment. In each session, students underwent all the treatments in Mayer's (1985) problem solving model, namely: problem translation, problem integration, solution planning, and solution execution.

Instrumentation

The research instrument used in this study is called as Mathematical Problem Solving Ability Test (MPSAT) for pre-test and post-test. MPSAT was adapted from Mathematical Processing Instrument (MPI) by Hegarty and Kozhevnikov (1999). In the pilot study done by Hegarty and Kozhnevnikov, the MPI gave internally consistent measures of problem solving success (Cronbach's a = .78) and solution strategy which is the tendency to use visual-spatial representations (Cronbach's a = .72). The items been revised for this study to improvise the grammar and to lengthier the questions/ added on some sentences which prompt students to draw when solving the questions. Also, the mathematical values (numbers) been changed in some items.

The instrument consisted of five items to gauge students' mathematical problem solving ability. All items were carefully designed to be solved by devising more than two strategies based on the system of coding in the MPSAT rubric, which also inclusive of visual representation strategy as one of the chosen strategy of solving the MPSAT questions. In addition, the MPSAT items were intentionally designed to align closely with the four steps of mathematical problem solving model as suggested by Mayer (1985). Each item of MPSAT was developed to collect students' responses so that the researcher will be able to determine students' overall problem solving ability and students' abilities to understand the problem, devise a plan, carry out the plan, and look back on the obtained solution prior and after the treatments.

Validity and Reliability

The instrument used to assess students' mathematical problem solving ability had undergone pilot study before it can be administered. According to Gay, Mills, and Airasian (2009), the validity and reliability of the instruments are two essential elements that must be carefully established in an instrument used in the research.

Content validity was chosen for this study as content validity of MPSAT refers to the degree that the MPSAT covers the content that it is supposed to measure. Two very experienced mathematics teachers were satisfied with the contents of MPSAT items which is in accordance with the Year 4 KSSR curriculum. They also found that the questions are suitable for the students' academic level and both reported that the MPSAT questions clearly assess the four steps of problem solving as stated by Mayer (1985).

On the other hand, two reliabilities namely the test-retest and inter-rater reliability were established for this study. There were 30 subjects in the sample ($n = 30$) participated and the test-retest results indicated that the MPSAT scores are significantly stable over time (Pearson's $r = .926$, $p = .000$), whereby a very high degree reliability was found between two raters (judges) scoring of the MPSAT using the MPSAT rubric. The average measure of Intraclass Correlation Coefficient (ICC) was .968 which means that the MPSAT rubric scores correlated 97% of the time.

Data Analysis

One-way ANCOVA test was performed to determine the significance of the mean difference between the MM group, MMVR group, and the control group on the mathematical problem solving performance outcome. One-way ANCOVA allows the researcher to statistically control for a third variable, which is sometimes known as a confounding variable, which may be negatively affecting the results (Pallant, 2010). In order to eliminate the threat of this inequality, the mean score of "Understand the Problem, "Devise a Plan", "Carry out the Plan", and "Looking Back" sections, and also the total of the overall mean score was calculated. According to the MPSAT rubric, "Understand the Problem" consists of three parts, whereas "Devise a Plan" and "Carry out the Plan" consist of two parts, respectively. "Looking Back" consists of four parts. The scoring was categorized into 4, 3, 2, and 1 points for each part.

FINDINGS

Mathematical Problem Solving Ability of Year 4 students after MM and MMVR treatments

To run the ANCOVA statistical analysis, a few assumptions need to be met. Figure 1 shows the scatterplot assessed to measure the assumptions of linearity, which was fulfilled, as there was a linear relationship between the posttest scores and pretest score as a covariate for control and experimental groups. Also, the boxplot in Figure 1 shows there are no outliers in data.

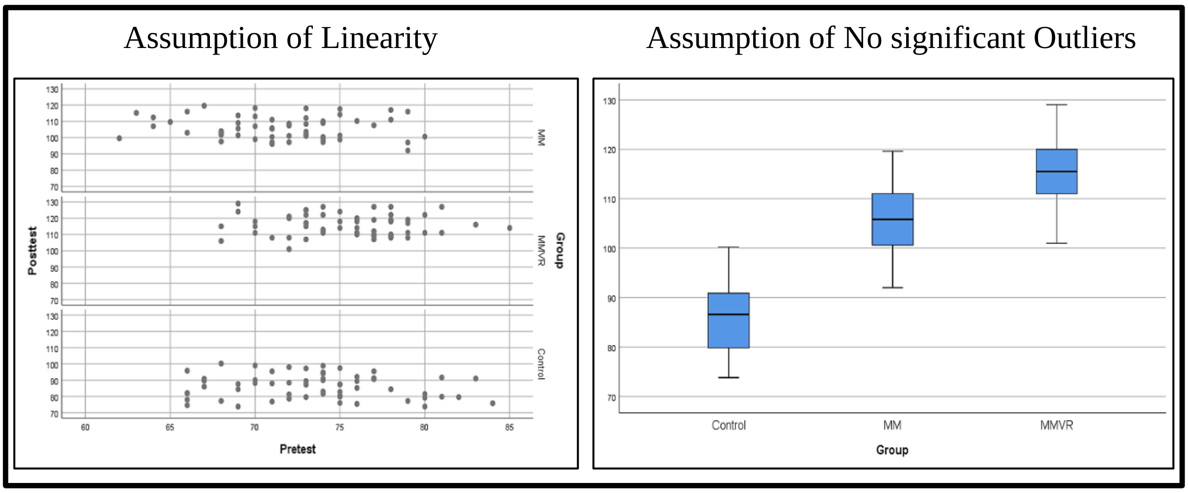

Figure 1. Assumptions of Linearity and Outliers

Table 2 below shows that there is a homogeneity of variance-covariance, as assessed by Levene's test of homogeneity of variance, $F(2, 172) = .66$, $p = .52$ ($p > .05$).

Table 2
Assumption of Homogeneity of Variance for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

F	*df1*	*df2*	*p*
.66	2	172	.52

There is a homogeneity of regression slopes as the interaction term was not statistically significant, $F(2, 169) = .18$, $p = .84$ ($p > .05$), as shown from Table 3 below.

Table 3
Assumption of Homogeneity of Regression Slopes for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

Source	*df*	*F*	*p*
Group * Pretest	2	.18	.84
Error	169		

Table 4 below shows the standardized residuals for posttest that were normally distributed as assessed by Shapiro-Wilk's test for MM, MMVR, and Control groups.

Table 4
Assumption of Normality for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

	Group	*df*	*p*
Standardized Residual for Posttest	MM	57	.065
	MMVR	58	.71
	Control	60	.096

As the required assumptions were met, the inferential analysis on scores were conducted. The following Table 5 and Table 6 show the mean differences of the posttest scores in MPSAT between the MM, MMVR, and Control groups. Figure 2 shows the adjusted means of posttest scores of MPSAT for MM, MMVR and Control groups.

Table 5
Tests of Between-Subjects Effects for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

Source	Type III Sum of Squares	*df*	Mean Square	*F*	*p*	Partial Eta Squared	Noncent. Parameter	Observed Power
Corrected Model	27604.624[a]	3	9201.541	194.533	.000	.773	583.598	1.000
Intercept	6768.232	1	6768.232	143.089	.000	.456	143.089	1.000
Pretest	952.851	1	2152.851	11.117	.000	.231	90.117	.883
Group	27354.876	2	13677.438	291.44	.000	.772	578.318	1.000
Error	8088.423	171	47.301					
Total	1871889.080	175						
Corrected Total	35693.048	174						

The results of the one-way ANCOVA indicates that there is a statistically significant difference in the mean of the posttest score in MPSAT between the three groups, $F(2, 171) = 291.44$, $p = .000$, with large effect size and strong power (partial $\eta_p^2 = .77$, observed power = 1). The effect size suggests that about 77% of the variation in posttest scores can be accounted for by the treatments in MM and MMVR groups. This conclude that Mayer's problem solving model teaching strategy treatment and Mayer's problem solving model with visual representation teaching strategy treatment have greatly improved Year 4 students' mathematical problem solving ability.

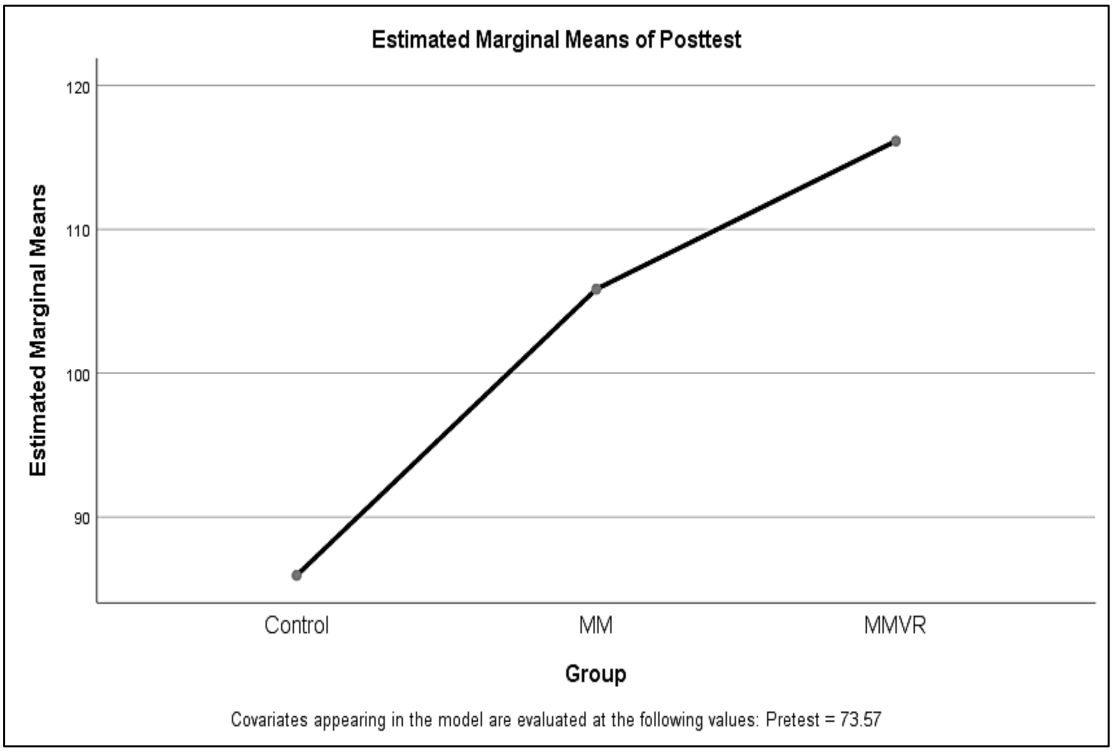

Figure 2. Estimated Marginal Means for Posttest Scores of MPSAT for MM, MMVR, and Control Groups

The adjusted means of posttest scores of MPSAT for MM, MMVR and Control groups were 105.84 (*SE* = .94), 116.14 (*SE* = .93), and 85.94 (*SE* = .89) respectively as shown in Figure 2 above. The adjusted mean of posttest scores for MMVR group was higher than the adjusted mean of posttest score of MM and control group respectively after the treatments.

Table 6
Post Hoc Analysis for Posttest Scores of Problem Solving Ability for MM Group, MMVR Group and Control Group

(I) Group	(J) Group	*MD* (I-J)	*p*
MM	Control	20.18	.000
	MMVR	-9.74	.000
MMVR	Control	29.92	.000
	MM	9.74	.000
Control	MM	-20.18	.000
	MMVR	-29.92	.000

Finally, post hoc analysis was performed with a Bonferroni adjustment as shown in the Table 6 above. The table gives a significant level for mean differences between MM, MMVR, and Control groups. There was a significant difference in posttest scores between MMVR and MM treatments, between MMVR treatment and Control group, and between MM treatment and Control group. The mean of posttest scores for the MMVR teaching strategy was different from the MM teaching strategy with 9.74, and was different than for the control group with 29.92. Meanwhile, the mean of posttest scores for MM teaching strategy was different than for the control group with 20.18.

DISCUSSION

The findings from this study showed that mathematical problem solving ability of Year 4 students in MMVR group has improved after students has undergone Mayer's problem solving model with visual representation teaching strategy treatment. The results of this study aligned with the findings of Ho and Lowrie (2014) stating that communication of mathematical ideas using visual such as simple picture aiding students in connecting ideas across the problem given, hence, improve the tackling technics of mathematical problems among students. Besides, to be a good problem solver, multiple representations should be able to be dealt with flexibly and switch adaptively between them (Acevedo Nistal, van Dooren, Clarebout, Elen, & Verschaffel, 2009; Dreher & Kuntze, 2015).

Students in MMVR group use drawing/ visual representation strategy from the first to fourth phase of Mayer's problem solving Model which are problem translation, problem integration, solution planning, and solution execution phases. Meanwhile, students in MM group only use drawing/ visual representation strategy in the first and second phases of Mayer's problem solving model which are problem translation and problem integration phases. This means, students in MM group were only taught by the teacher to use drawing strategy in understanding the problem story stage. During the post MPSAT session, many students in MMVR group used visual representation method in problem translation, problem integration, solution planning, and solution execution phases as taught to them during the previous ten treatment sessions. From the posttest result of MPSAT, students in MMVR group who solved the MPSAT questions using visual method tend to achieve more accurate solutions for the given problems compared to students in MM group and Control group.

This adds to the work completed by Ho and Lowrie (2014) which reported that students preferred to use visual method more when solving difficult problems. Besides, one of the issue found by many students at different stages was that visualization alone created too much 'clutter' in their brain and that drawing helped to 'free up space'(Teahen, 2015). This ties in with theories that working memory has a capacity and can only retain so much information (Cowan, 2014). Trying to create a mental image, figuring out the equation and then calculating the equation seemed too much to keep in the working memory for some students. Feedback from students indicated that using drawings to get some of the information down or to help with the calculation of the problem enables the students to be less stressed and be more accurate in their working.

In this study, as discussed earlier, students were encouraged to use visual representation strategy from the first to fourth phase of Mayer's problem solving Model which are problem translation, problem integration, solution planning, and solution execution phases. This teaching method shows the possibility that students' ability to engage in the process of relating and translating information when dealing with representations is governed by the type of strategy taught by teachers in solving mathematical problems.

CONCLUSION

In this study, Mayer's problem solving Model with Visual Representation (MMVR) teaching strategy has proven to be an effective tool in enhancing students' mathematical problem solving ability. The learning process in this study made students actively participate in understanding and solving the mathematical problems with the help of visual representation strategy. The visualization strategy is easy for students to use as it is often something they are encouraged to do when reading. This study shows that a major difference can be made with a simple strategy without negative consequences for teachers or students.

It may be reasonable to encourage existing math teachers to share and learn together on how to implement Mayer's problem solving Model with visual representation teaching strategy to improve students' mathematical problem solving ability in their schools. Also, curriculum developers should take into consideration to develop a study materials includes combination of Mayer's problem solving Model with visual representation instruction focusing on traditional textbook-based math skills with project-based problem solving activities in order to improve both specific and generalized problem solving ability.

While this study was limited by the use of small sample size and focussing only on the Year 4 students with arithmetic topic, future studies can be conducted by testing this strategy on a larger sample size. The research can also consider different school grades with focusing on different topics using Mayer's problem solving Model with Visual Representation teaching strategy.

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