

Students' Knowledge in Representation in Learning Mathematics: Establishing Validity in A Pilot Test

Nur Ikhwany Hj Kamaruddin*, Teoh Sian Hoon and Joseph Boon Zik Hong

Department of Mathematics, Faculty of Education, Universiti Teknologi MARA,
42300 Bandar Puncak Alam, Selangor, Malaysia

*corresponding Author: nikhwany@gmail.com

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Abstract

The cognitive process of representation, which is key to learning mathematics, is crucial. However, fewer guidelines are given. Since representation plays a complex role in cognition, it is essential to gather data using a qualitative approach. As a result, the purpose of this study is to conduct a pilot test to establish the credibility and dependability of research. In the pilot test, a set of interview questions was tested, focusing on the investigation of how students acquire mathematics through representation. Specifically, the objective of this study is to pilot test the instrument through interview session. The pilot test was conducted on three primary school pupils as groundwork for thesis on representation in fraction and related to pupils' metacognition skill. The findings from this study are that several questions in the instrument need to be modified according to the responds of the pupils. From this study, it was recommended that a pilot test is needed to be done on the questions in the instrument through several interview session to make sure that the instrument questions can achieve to answer the objective of the study.

Keywords: Pilot Study, Representation, Interview, Mathematics, Qualitative

Introduction

Learning mathematics via representation is commonly practised, nevertheless it is limited to enactive, iconic, and symbolic (Bruner, 1966). Enactive refers to the action learning, meanwhile iconic is the learning that uses visualization of diagrams, images, and pictures and symbolic refers to learning with the use of symbols and abstract situations related to mathematics. Before student make steps to iconic and symbolic representation, they should initially have the chance to work actively with concrete objects. Over the past three decades, mathematics learning has been focusing on the use of representations to develop the abilities of the pupils in utilising the suitable representations. The key factor to be good in both mathematical thinking and problem solving is the skill to be flexible with several types of representations and able to change between representations. The essential of mathematics concept is the process of making connections and conversions between diverse types of representations.

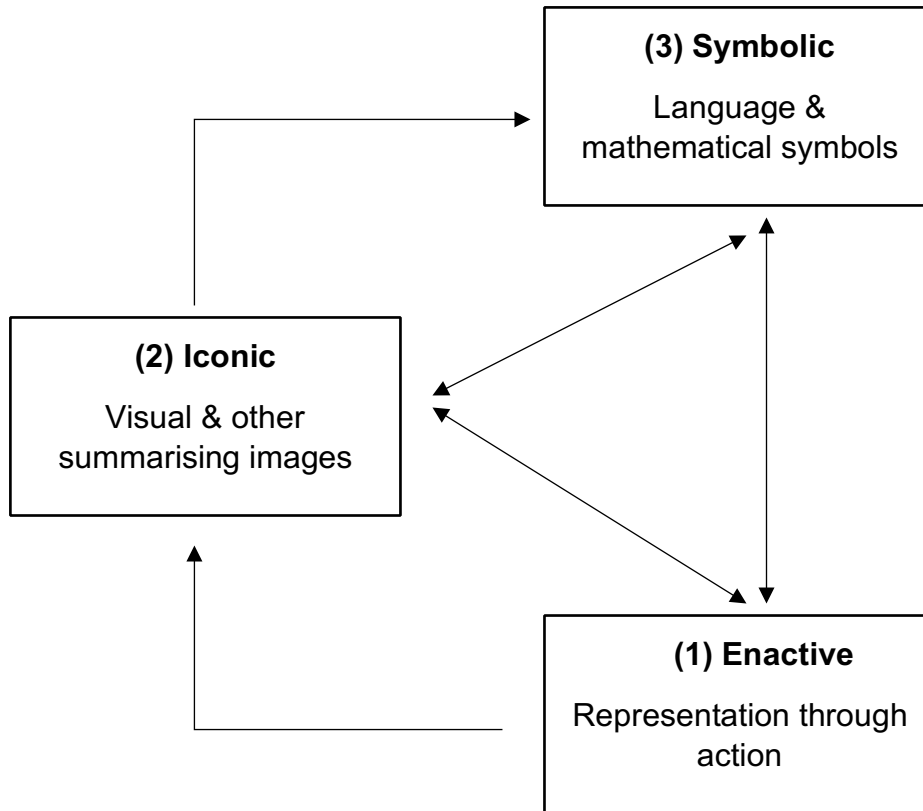


Figure 1: Bruner's (1966) Three Modes of Representation

In monitoring whether students can recognize representation and knows well about it, metacognition is involves. Metacognition control the process of learning and control the thinking. In the primary school, problem solving seems to be not so important but mathematical thinking is important. This is because students need to adapt to control their thinking.

Metacognition is not a straightforward process to be study, but it is possible to be known, through a qualitative approach that is interview. Thus, this study is trying the best to develop and validate interview questions as the instrument focusing on the metacognition skills of the students in understanding representation of enactive, iconic, and symbolic.

This study is design by implementing explanatory sequential design which is applying both quantitative and qualitative method to answer all research questions. Explanatory sequential research design is to use a qualitative strand to explain initial quantitative results (Creswell, 2013). For this recent paper, it was conducted at the preliminary stage of qualitative phase in developing the instrument to measure the metacognition skill of the students in understanding representation.

Methodology

To investigate on students' understanding and knowledge of representation focusing on enactive, iconic, and symbolic (Bruner, 1966), a qualitative approach was applied. There are numerous studies that used qualitative approach in knowing more and in depth on certain issues by using the interview approach. The data collected using interviews allow the researcher to view and understand the representation phenomenon from the student's perspective (Merriam, 2009). It is different from quantitative approach which the researcher acts as the primary instrument of the data. Furthermore, to seek insights of students that have learned on representation, the suitable technique or approach is a one-to-one interview of clinical interview. A clinical interview is where the one-to-one interview was being held for more than one time on the same respondent until the objective of the interview was accomplished (diSessa, 2007). To do so, before conducting a clinical interview, it is particularly important to pilot testing the interview session to gain some practice in interviewing which would involves probing during the interview session.

To test a particular research instrument, pilot studies are commonly connected with quantitative approach (Van Teijlingen & Hundley, 2002). For a major study, preparation is needed which includes the pilot work that originate from the qualitative inquiry. Certain research procedures including sample size and selection, and test the measurement instrument is needed to consider during the pilot testing which will be used to report the possible practical issues that might be occurred during the interview sessions. Thus, based on the references that been found, a pilot work for the interviews is fairly needed.

Through the piloting of interviews, the interview protocol itself could be reinforced resulting from the data and process of pilot work from the interview sessions (Castillo-Montoya, 2016). Required adjustments and changes to the major study would be acceptable and the restrictions within the interview design can be recognized from the piloting of interviews (Kvale, 2007). But, in the literature, the process of pilot work was ineffectively informed and described (Van Teijlingen *et al.*, 2001). As the interview sessions developed, it would improve the quality of the interview, which make the necessity for piloting the qualitative interviews to be not noticeable. The pilot study is essentials to assist the major study before going further into the qualitative data and support in regulating the interview direction, (Harding, 2013). To provide consistency of data, establishing the interview direction is very crucial because it would assist the qualitative data, particularly to answer the researcher's objectives of the study (Krauss *et al.*, 2009).

Pilot study

In this study, several steps in conducting the pilot test for interviews to answer the research questions of the study has been conducted by the researcher. Figure 2 below shows the process that involve in the pilot work for interviews. Every step in the process is discussed based on how it helps the researcher to develop the interview

process to be smooth with the presence of probing. These steps were adapted from Abdul Majid *et.al* (2017) study on pilot study for interviews.

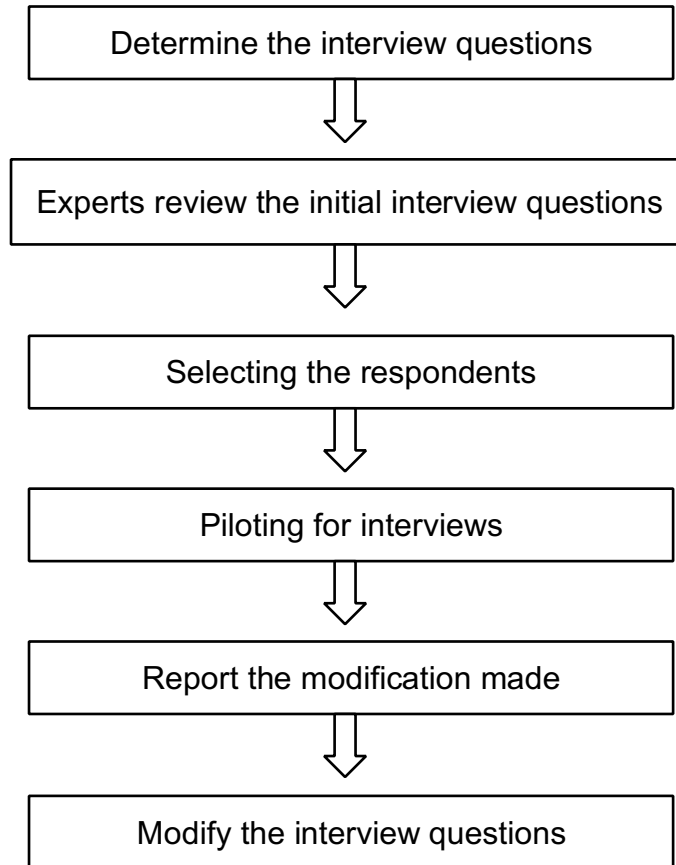


Figure 2: Steps in Conducting the Pilot Study for Interviews

Determine the interview questions

The interview session was guided with open-ended questions that is probed from the answer of the students while they are answering the representation questions. Researcher focussed on making sure that the questions for the interview are created to answer the research question. The effectiveness of the interview questions can be increased from this pilot test and the consequence to accomplish the purpose of this study can also be ensured (Castillo-Montoya, 2016). The interview questions have been confirmed by the researcher that it includes all the needed questions that would assist in measuring the metacognition of the students. The questions for the interview are open-ended questions and been checked by the experts.

Experts review the initial interview questions

Next, the experts of the field have revised the interview questions relating to the relevance of the questions, language and the wording use that is suitable for primary school students. The interview questions contained of fundamental questions that was tested in the pilot test and probing questions was also used by the researcher to explore more on students' metacognition skills in understanding enactive, iconic, and symbolic representation.

Selecting the respondents

Respondents were selected from one primary school which involves Year 3, Year 4, and Year 5 pupils. Permission both from the Head Mistress of the school and the pupils was gained to make the pilot work to be run smoothly. The respondents for a pilot work should have the same criteria as possible with the respondents of the major study because it would picture the similar responds from both respondents which would help in the pilot work (Turner, 2010). Table 1 below shows the demographic of the respondents that contribute to the pilot work.

Table 1: Demographic of The Respondents for Pilot Test

<i>Pseudonym / Nickname</i>	<i>Year</i>	<i>Gender</i>
Nur	3	Female
Jess	4	Female
Kajol	5	Female

Piloting for interviews

The pilot study was conducted in May and June 2022. The interview session was held in the 'Bilik Cakna' of the school to have a quiet and focus place for students to answer the interview questions. The interview session was held for about 10 to 20 minutes per respondents and was recorded using the researcher's phone. This pilot study was aiming to test the suitability of the interview questions and gave the researcher some ideas to improve on the practicability of the research itself.

Report the modification made

Once the pilot study has completed, the interview was transcribed by the researcher based on the recorded audio and code the data with the identify codes. Here, the strategies have been refined from the researcher based on the outcome of the pilot test conducted. From the pilot test, it was found that the age of the respondents to be selected for the main study are Year 4 and Year 5 pupils. On top of that, the way of probing the interview questions also need to be modified according to the diverse types of representation because each representation has different approach in probing the questions to achieve the objectives.

Results and Discussion

Table 2 below shows the findings from the pilot test.

Table 2: Findings from the pilot test

<i>Respondent</i>	<i>Interview</i>	<i>Findings</i>
Nur (Y3) (Enactive)	<p>I: How do you determine $\frac{1}{3}$ of the chocolate bar given?</p> <p>R: For (a), I can just easily find $\frac{1}{3}$ of the chocolate since the chocolate is divided into 3 equal parts.</p> <p>I: Good, how about (b)?</p> <p>R: Ermm, for chocolate (b), I count the chocolate bar that been divided first, then I divide into 3 because $\frac{1}{3}$.</p> <p>I: Can you show me how you count it?</p> <p>R: I count and find out that total small square of the chocolate is 6 vertical rows. When divide into 3, I get 2 vertical rows of the chocolate that represent $\frac{1}{3}$ of it.</p> <p>I: Okay, good one. But for (c), since there is no small square on the chocolate, how to find the answer?</p> <p>R: Yes teacher, since there are no small squares, I just divide the chocolate bar into 3 equal parts and shade one part of it.</p> <p>I: Can you show me how?</p>	<p>From the interview, respondent is aware of how $\frac{1}{3}$ looks like when given a concrete picture that they have seen daily which is chocolate bar. Respondents understand and have knowledge on how to divide the chocolate bar into three parts since the fraction is $\frac{1}{3}$. Although given three different types of chocolate bar, respondent know how to shade $\frac{1}{3}$ of the chocolate bar.</p>
Kajol (Y5) (Enactive)	<p>I: How do you know that $\frac{1}{4}$ of the cookies are 2 from the total number of cookies?</p> <p>R: First I count the total number of cookies and get 8 cookies altogether.</p> <p>I: Good, then what did you do?</p> <p>R: Then, I divide the cookies into four parts that is equal.</p>	<p>This question is the continuous question that test for the respondent knowledge of fraction part whole that is similar with the question on chocolate bar. Kajol demonstrates that she knows how to count the cookies to know what is $\frac{1}{4}$ of the cookies. She understands that the total cookies</p>

	<p>I: Why you divide into 4 equal parts?</p> <p>R: Because.... the question asks for 1 over 4 of the total cookies, so 4 equal parts, then I get 2 in each part. So, the answer is 2 cookies.</p> <p>I: Okay that is true. Is there any other easier way that you can do to find the answer?</p> <p>R: I just multiply $\frac{1}{4}$ with 8 and get 2 as the answer, teacher.</p> <p>I: I see, that also get the same answer, right?</p> <p>R: Yes, teacher.</p>	<p>need to be divided into 4 equal parts to know the answer. Since she had learnt on fraction from a value, then she also explains that by multiply $\frac{1}{4}$ with the total number of cookies that are 8 can easily get 2 to be the answer.</p>
Nur (Y3) (Iconic)	<p>I: Girl, why did you choose triangle and square to be the answer for this question?</p> <p>R: I need to find the shapes that have $\frac{1}{2}$.</p> <p>I: Good, then how do you choose the right shapes to be the answer?</p> <p>R: For $\frac{1}{2}$, I need to find the shape that shows half of the shape. So, for this question, the shape that is shaded half is triangle and square.</p> <p>I: What about the blue rectangle and circle?</p> <p>R: Urrm the blue rectangle does not shade $\frac{1}{2}$ and the circle is shade not half of it.</p> <p>I: I see, thank you Nur. What can you do to make it half shaded?</p>	<p>From the interview, respondent recognize that $\frac{1}{2}$ is half. Thus, to find the shape that shows $\frac{1}{2}$ as the shaded part, she identifies the shaded part of the given 4 different shapes that shows half.</p>

Jess (Y4) (Iconic)	<p>I: How do you solve this question? R: Since it was given circle to be shade based on the given question, so I need to shade the circle and find the answer.</p> <p>I: That is true, but how do you shade the circle? R: The first circle I need to shade $\frac{5}{6}$. I divide the circle to 6 parts that is same size, and shade 5 parts.</p> <p>I: Good, but how about the circle for $\frac{1}{3}$? R: To find the answer, I need to make sure the denominator is the same, so for $\frac{1}{3}$, I need to change the denominator to 6 by multiply with 2, so I get $\frac{2}{6}$ and now I divide the circle into 6 parts and shade 2 parts.</p> <p>I: So, from the circles that been shaded, can now you find the answer for the equation? R: Yes teacher, I just subtract 5 with 2 and get 3. So, the answer is $\frac{2}{6}$ and divide the circle into 6 parts same as before and shade 2 parts for answer.</p> <p>I: Good one.</p>	<p>This question test on respondent's understanding on fraction addition. Respondent shade the first circle $\frac{5}{6}$ without any problem, then for the second circle, she changes the denominator to be the same as the first fraction because to add two fractions, they must have same denominator. Then, she continues to shade the second circle with $\frac{2}{6}$. To calculate for answer, she minus the shaded part and get the answer of $\frac{3}{6}$ to be shaded. Here, it portrays that the respondent has good understanding of fraction addition.</p>
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Jess (Y4) (Symbolic)	<p>I: For this question, how do you find the answer? R: For this question, I read and find all the information to find the answer.</p> <p>I: Good, then what did you do? R: I use the information given to calculate for the answer. I add $\frac{2}{5}$ with $\frac{1}{4}$ since the question ask for</p>	<p>From this interview, the respondent is aware on how to answer and solve a problem-solving question. She reads the question carefully and identify the information. She understands that she needs to add both fractions given to answer the question.</p>
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the total lawn that been mowed by John and his brother.

I: That is good. Show how you add the fractions.

Modify the interview questions

The interview questions were then modified based on the outcome of the pilot test. The modified interview questions were then checked and approved by the experts of the field. Finalize interview questions that been achieved from this pilot work will be used to collect data for the main study. Table 3 below shows the suggested modification been made for the next interview session.

Table 3: Modification for interview questions

<i>Respondent</i>	<i>Findings</i>	<i>Suggested Modification</i>
Nur (Y3)	From the interview, respondent is aware of how $\frac{1}{3}$ looks like when given a concrete picture that they have seen daily which is chocolate bar. Respondents understand and have knowledge on how to divide the chocolate bar into three parts since the fraction is $\frac{1}{3}$. Although given three diverse types of chocolate bar, respondent know how to shade $\frac{1}{3}$ of the chocolate bar.	<p>Researcher can continue to interview the same respondent on question that is similar but using other daily objects that they have seen but use the approach where they need to count the objects. For this study, the next question in the instrument measure on this.</p> <p>Q: $\frac{1}{6}$ of the dumplings were taken away. Colour the number of dumplings that has been taken away.</p>  <p>R: How about this question, how do you find $\frac{1}{6}$ of the dumplings?</p>

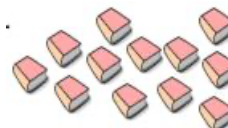
Kajol (Y5) This question is the continuous question that test for the respondent knowledge of fraction part whole that is similar with the question on chocolate bar. Kajol demonstrates that she knows how to count the cookies to know what is $\frac{1}{4}$ of the cookies. She understands that the total cookies need to be divided into 4 equal parts to know the answer. Since she had learnt on fraction from a value, then she also explains that by multiply $\frac{1}{4}$ with the total number of cookies that are 8 can easily get 2 to be the answer.

The question related needs to be asked more than one time to see the respondents' understanding of fraction in counting the objects. The next 3 questions in the instrument are like this but with different view.

Q: Circle $\frac{2}{3}$ of the total number of cars.



Q: Circle $\frac{3}{4}$ of the total number of books.



Q: Circle $\frac{3}{4}$ of the total number of carrots.




Nur (Y3) From the interview, respondent recognize that $\frac{1}{2}$ is half. Thus, to find the shape that shows $\frac{1}{2}$ as the shaded part, she identifies the shaded part of the given 4 different shapes that shows half.

To go in depth, researcher can continue to ask the respondent about the other two shapes. R: Why do you think that the other two shapes are not shaded half or $\frac{1}{2}$ of it?

R: If so, then show me how to make it shaded $\frac{1}{2}$ of its shape.

The next question in the instrument is similar, so researcher can ask related questions that lead the respondent to explains and prove their answer.

Jess (Y4)	This question test on respondent's understanding on fraction addition. Respondent shade the first circle $\frac{5}{6}$ without any problem, then for the second circle, she changes the denominator to be the same as the first fraction because to add two fractions, they must have same denominator. Then, she continues to shade the second circle with $\frac{2}{6}$. To calculate for answer, she minus the shaded part and get the answer of $\frac{3}{6}$ to be shaded. Here, it portrays that the respondent has good understanding of fraction addition.	Researcher can investigate more on this question by giving the respondent to shade on other shape that the researcher draws to test their understanding to shade fractions. R: What if I have the shape of a rectangle, how would you shade $\frac{5}{6}$ out of it?  The next question also tests the same on fraction addition but using different fractions, so researcher can apply the same approach to probe the interview questions to the respondent.
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Jess (Y4)	From this interview, the respondent is aware on how to answer and solve a problem-solving question. She reads the question carefully and identify the information. She understands that she needs to add both fractions given to answer the question.	For symbolic representation questions, researcher can ask the respondent on each step of calculation to know in depth how they visualize fractions during addition, subtraction, and multiplication.
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Based on the pilot test result, certain modification has been made in terms of the way the interview to be done by taking into place on the probing questions that leads to the answer that would reflect the research objectives of the study. Probing questions is important in an interview because it will assist in finding the outcome of the study (Moerman, 2014). Besides that, the respondents that will be selected in the study will be Year 4 and Year 5 pupils based on the interview result from the pilot test. The criteria of the respondents that will be involved in the study does impact the nature of methodology of the study (Mohajan, 2018).

Conclusion

This pilot study has a limitation of small number of respondents that specify that the data capacity was not reached. In order to find information that would help to explore and improve more on the qualitative data of the main study, this minimal pilot test was carried out with the hope that it will help to improve the main study outcome. Suggestions from this pilot study was the suitable respondents would be chosen for the main study and also the appropriate interview questions were chosen to be asked to the respondents. Recommendations that can be made here includes constructing the same research in a new context within the different settings for secondary school students.

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